Student Solutions Manual

containing selected solutions to

FUNDAMENTALS OF MACHINE COMPONENT DESIGN

3^{RD} Edition

by

Robert C. Juvinall and Kurt M. Marshek

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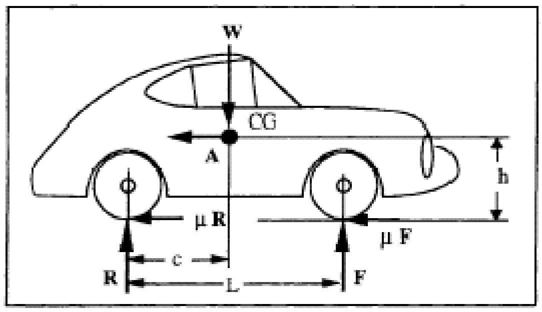
SOLUTION (2.3)

Known: An automobile of weight W and wheel base L slides while braking on pavement with given coefficient of friction. The location of the center of gravity is specified.

Find: Draw a free-body diagram of the automobile.

Schematic and Given Data:



Assumptions:
1. The friction force is constant during braking.
2. The vehicle deceleration is uniform.
3. The motor exerts negligible torque on the wheels (the motor is disconnected).

Analysis:
1. From summation of moments at point R,
   $\sum M_R = -F(L) - Ah + Wc = 0$

2. From summation of forces in the vertical direction,
   $\sum F_y = 0 = R - W + F = 0$

3. From summation of forces in the horizontal direction,
   $\sum F_x = 0 = A + \mu R + \mu F = 0$

4. Solving by eliminating A and R gives
   $-FL + \mu hW + Wc = 0$

5. Solving for F gives, $F = \dfrac{W(c + \mu h)}{L}$

Comment: For a 4000 lb vehicle, with L =120 in., h = 26 in., c = 70 in., and $\mu = 0.7$:

1. $F = 4000\text{ lb}\,\dfrac{[70\text{ in} + (0.7)(26\text{ in})]}{120} = 2940\text{ lb}$

2. With the vehicle stationary, the static force on the two front tires is 2330 lb.

SOLUTION (2.9)

Known: An engine and propeller rotate clockwise viewed from the propeller end. A reduction gear housing is bolted to the engine housing through the bolt holes shown. The power and angular velocity of the engine are known.

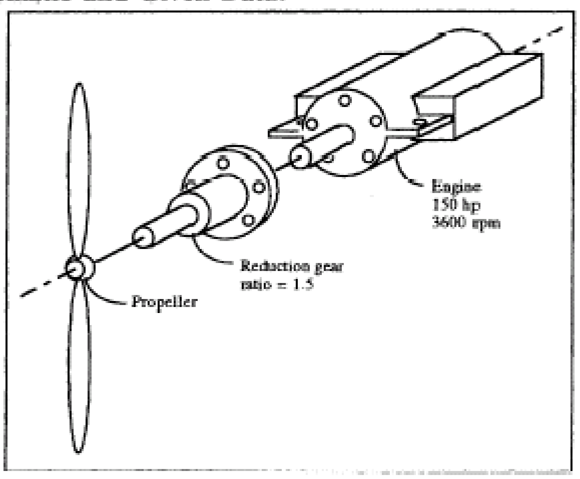
Find:

(a) Determine the direction and magnitude of the torque applied to the engine housing by the reduction gear housing.

(b) Determine the magnitude and direction of the torque reaction tending to rotate (roll) the aircraft.

(c) Find an advantage of using opposite-rotating engines with twin-engine propeller-driven aircraft.

Schematic and Given Data:

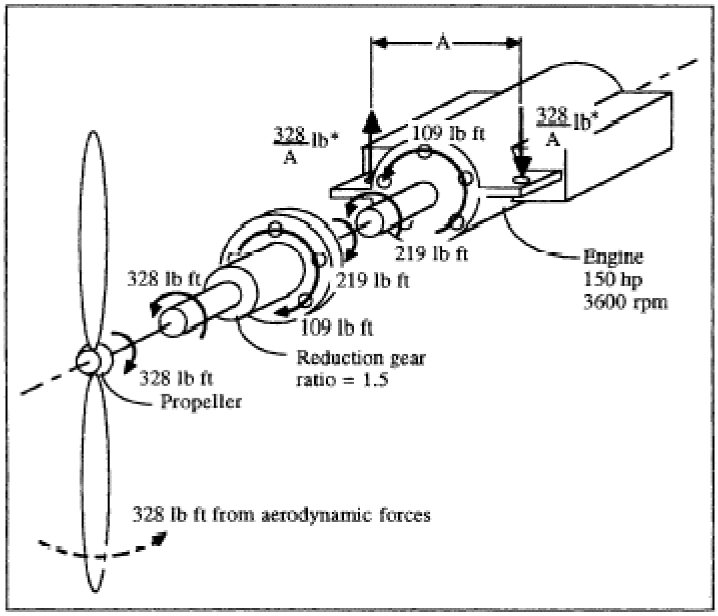


Assumption: The friction losses are negligible.

Analysis:

1. From Eq. (1.3), engine torque, $T = \frac{5252 \cdot \dot{W}}{n} = \frac{5252(150)}{3600} = 219$ lb ft

2. Reduction gear torque, $T = 219(1.5) = 328$ lb ft



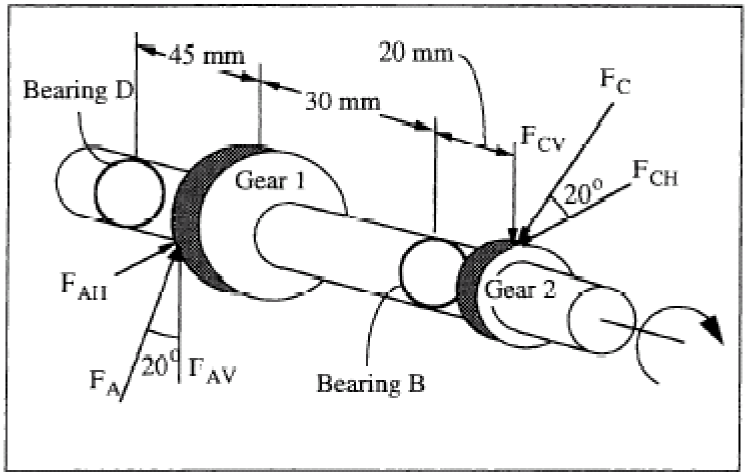
3. The attachment forces apply an equal and opposite torque of 328 lb.ft ccw tending to "roll" the airplane--see (*) in the above figure.
4. Thus, the torque applied to the engine housing by the reduction gear housing is 109 lb ft counter-clockwise, and the torque reaction tending to rotate the aircraft is 328 lb ft counter-clockwise. ■
5. Torque reactions applied to the air frame by the two engines cancel. (This produces bending in the connecting structure, but does not require a compensating roll torque from the aerodynamic control surfaces.) ■

SOLUTION (2.17)

Known: The geometry and dimensions of the gear and shaft assembly are known.

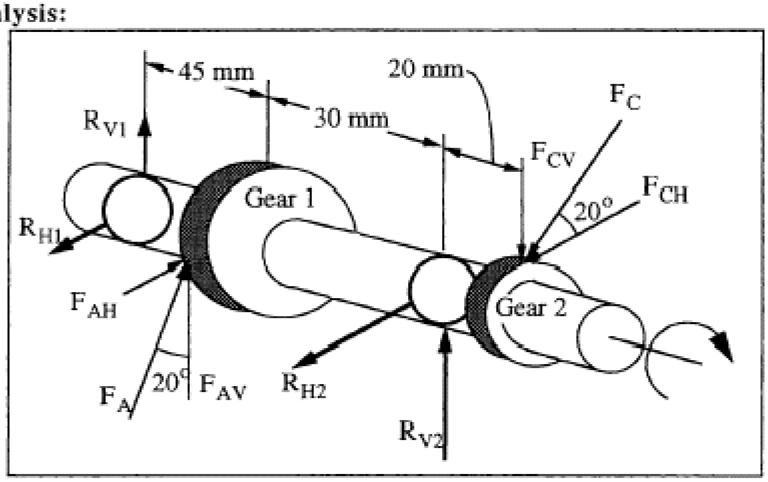
Find: Draw a free-body diagram of the assembly. Also draw the free-body diagrams for gear 1, gear 2 and the shaft.

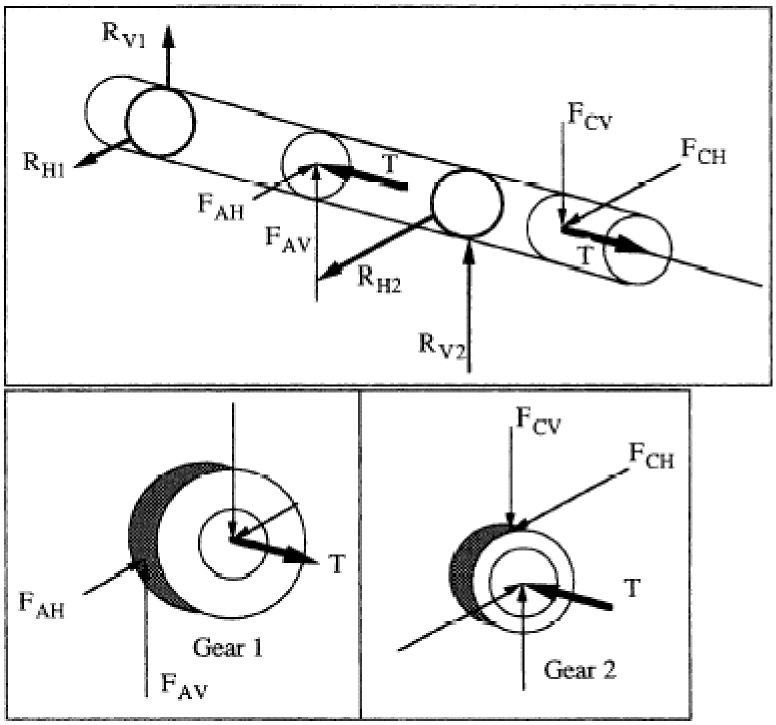
Schematic and Given Data:



Assumption: Gravity forces are negligible.

Analysis:





R_{V1}
R_{H1}
F_{CV}
F_{CH}
T
F_{AH}
F_{AV}
R_{H2}
T
R_{V2}
T
F_{AH}
Gear 1
F_{AV}
F_{CV}
F_{CH}
T
Gear 2

SOLUTION (2.27)

Known: A force P is applied to an engine crankshaft by a connecting rod. The shaft is supported by main bearings A and B. Torque is transmitted to an attached member through flange F.

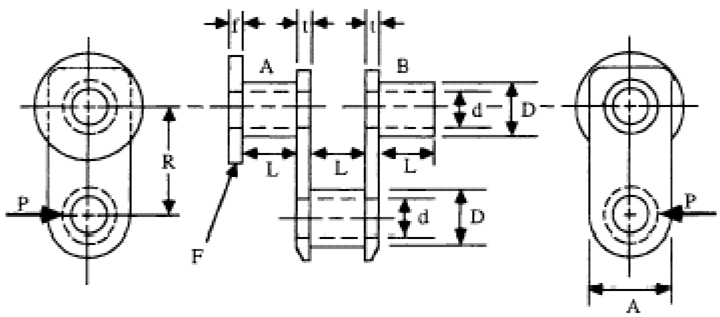
Find:

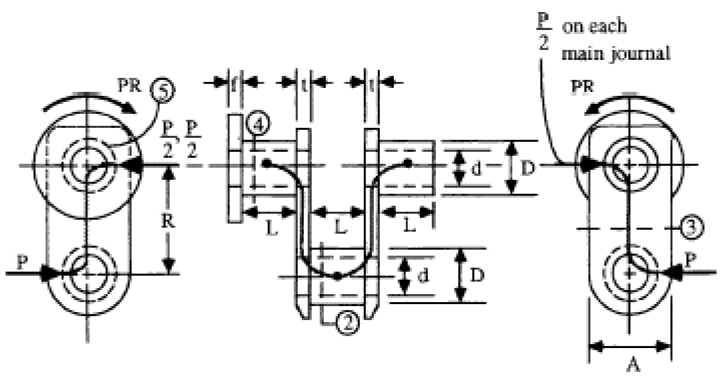
(a) Draw the shaft, and show all loads necessary to place it in equilibrium as a free-body.

(b) Starting with P and following the force paths through the shaft to the flange, identify the locations of potentially critical stresses.

(c) Making appropriate simplifying assumptions, write an equation for each.

Schematic and Given Data:





Analysis:

1. Where "P" is applied to the crankpin, the compressive stress (assuming uniform stress distribution) is given by:

$$\sigma = \frac{P}{\text{Projected Area}} = P/DL$$

2. The shear stress at section 2 (assuming uniform stress distribution) is:

$$\tau = \frac{P}{2\pi(D^2 - d^2)/4} ; \quad \tau = \frac{2P}{\pi(D^2 - d^2)}$$

3. The shear stress at section 3 (assuming a uniform distribution): $\tau = P/2tA$
4. The torsional stress at section 4 (neglecting stress concentration):

$$\tau = \frac{Tc}{J} = \frac{(PR)(D/2)}{\frac{\pi}{32}(D^4 - d^4)} = \frac{16PRD}{\pi(D^4 - d^4)}$$

5. The shear stress at cylindrical section 5:

$$\tau = \frac{T}{\pi Df(D/2)} = \frac{2PR}{\pi D^2 f}$$

6. Bending stresses are also present, the magnitudes of which depend on rigidities of the shaft and associated components, and on the fits between these components.

SOLUTION (2.31)

Known: Two plates are joined with straps and a single row of rivets (or bolts). Plates, straps, and rivets are made of ductile steel with known yield strengths.

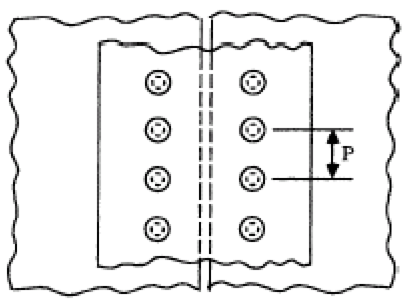
Find:

(a) Calculate the force F that can be transmitted across the joint per pitch P, of joint width, based on the rivet shear strength.

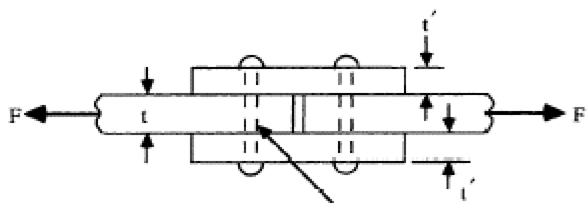
(b) Determine minimum values of t, t′, and P that will permit the total joint to transmit this same force (thus giving a balanced design).

(c) Determine the efficiency of the joint (ratio of joint strength to strength of a continuous plate).

Schematic and Given Data:



$S_{yt} = 300$ MPa
$S_{yc} = 300$ MPa
$S_{ys} = 170$ MPa



Assumption: The frictional forces between the plates and straps are negligible.

Analysis:

(a) Each pitch involves transmitting force "F" through 1 rivet in double shear:

$$F = 2\left(\frac{\pi d^2}{4}\right) \bullet S_{ys} = 2\ (25\pi\ \text{mm}^2)(170\ \text{MPa}) = 26{,}700\ \text{N} \qquad \blacksquare$$

(b) For plate and strap to have equal tensile strength and equal compressive strength (at rivet interface), t = 2t′.

The compressive load carrying capacity (at rivet interface) is F = Projected area • S_{yc} :

26,700 N = 10 t mm² • 300 MPa. Hence, t = 8.90 mm; t′ = 4.45 mm. ■

The tensile load carrying capacity (at rivet interface) is F = (P -10) t • S_{yt} :

26,700 N = (P - 10)(8.90) mm² • 300 MPa

P = 20 mm ■

(c) $$\text{Efficiency} = \frac{\text{Joint strength}}{\text{Strength of a continuous plate}} = \frac{26{,}700}{S_{yt}(t)(P)} =$$

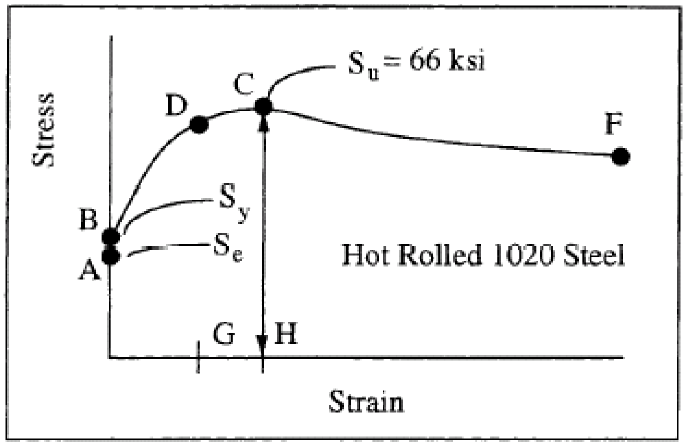
$$\frac{26{,}700}{(300\ \text{MPa})(8.90\ \text{mm})(20\ \text{mm})} = 0.50 = 50\% \qquad \blacksquare$$

SOLUTION (3.5)

Known: A tensile specimen of a known material is loaded to the ultimate stress, then unloaded and reloaded to the ultimate stress point.

Find: Estimate the values of σ, ε, σ_T, ε_T for the first loading and the reloading.

Schematic and Given Data:



Assumption: After unloading the stress-strain curve starts at point H for the new specimen.

Analysis:

1. For the initial sample, σ = 66 ksi, ε = 30%. ■
2. For Figure 3.2, R = 1.3 at point H.
3. From Eq. (3.4), $\sigma_T = \sigma R$ = (66)(1.3) = 85.8 ksi. ■
4. From Eq. (3.5), $\varepsilon_T = \ln(1 + \varepsilon) = \ln 1.30 = 0.26$ = 26%. ■
5. For the new specimen; σ = 66(1.3) = 85.8 ksi. ■
6. The new specimen behaves elastically, so $\varepsilon = \sigma/E$ = 85.8/30,000 = .00286. ■
7. Within the elastic range, $\sigma_T \approx \sigma$ and $\varepsilon_T \approx \varepsilon$. Therefore σ_T = 85.8 ksi and ε_T = 0.29%. ■

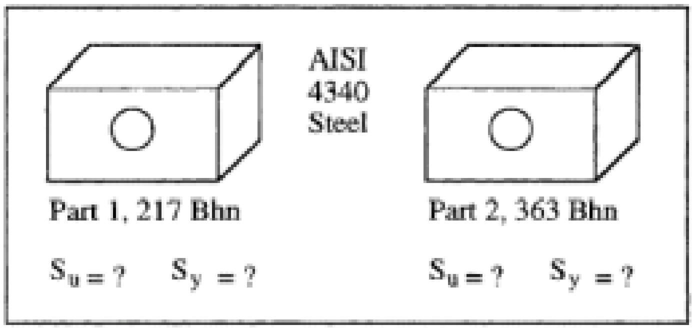
Comment: Note also that $\varepsilon_T = \ln(1 + \varepsilon) = \ln(1.0029)$ = 0.29%.

SOLUTION (3.11)

Known: An AISI 4340 steel part is heat treated to 217 Bhn. A second AISI 4340 steel part is heat treated to 363 Bhn.

Find: Estimate values of S_u and S_y for both parts.

Schematic and Given Data:



Assumptions:
1. The experimentally determined relationship of ultimate strength to hardness is sufficiently accurate.
2. The experimentally developed relationship of yield strength to ultimate strength is sufficiently accurate for our purposes.

Analysis:
1. Using Eq. (3.11), we can estimate S_u.
   $S_u = K_B H_B$
   where $K_B = 500$ for most steels.
   For Part 1, $S_u = 500(217) = 108{,}500$ psi; for Part 2, $S_u = 500(363)$ $= 181{,}500$ psi ■
2. S_y can be estimated by using Eq. (3.12). $S_y = 1.05\ S_u - 30{,}000$ psi. For Part 1, $S_y = 1.05(108{,}500) - 30{,}000 = 83{,}925$ psi. For Part 2, $S_y = 1.05(181{,}500) - 30{,}000 = 160{,}575$ psi. ■

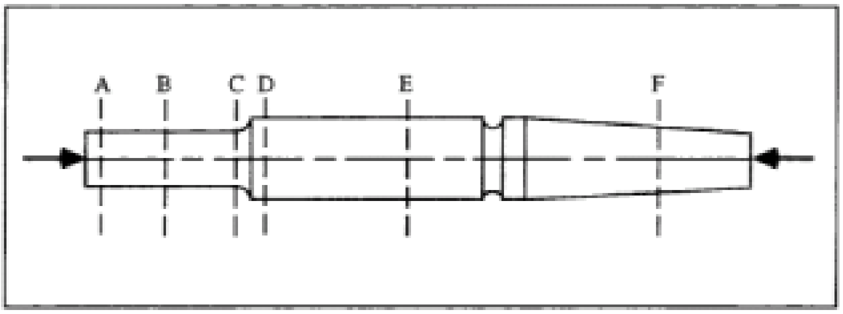
Comments:
1. Equation (3.12) is a good estimate of the tensile yield strength of stress-relieved (not cold-worked) steels.
2. Experimental data would be helpful to refine the above equations for specific steels.

SOLUTION (4.2)

Known: A shaft is axially loaded in compression.

Find: List the lettered sections where the average compressive stress is equal to P/A. Also list the sections where the maximum stress is equal to P/A.

Schematic and Given Data:



Assumptions:

1. The shaft will not buckle.
2. There are no residual stresses in the shaft.
3. The load is applied exactly along the centroidal axis of the bar.

Analysis:

1. All sections have average stress = P/A.
2. Sections B, E, and F should have maximum stress approximately equal to P/A.

Comment: The maximum stress is equal to P/A only if the load is uniformly distributed over the cross section.

SOLUTION (4.5)

Known: A steel propeller shaft with a given diameter transmits a known power at a specified angular velocity.

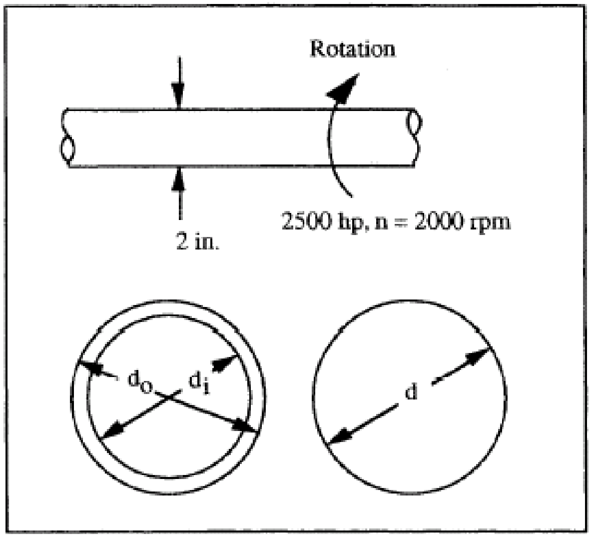
Find:

(a) Determine the nominal shear stress at the surface.

(b) Determine the outside diameter required to give the same outer surface stress if a hollow shaft of inside diameter 0.9 times the outside diameter is used.

(c) Compare the weights of the solid and hollow shafts.

Schematic and Given Data:



Assumptions:

1. Bending and axial loads are negligible.
2. The bar is straight and round.
3. The material is homogeneous, and perfectly elastic within the stress range involved.
4. The effect of stress raisers is negligible.

Analysis:

1. From Eq. (1.3), $T = \dfrac{5252\,\dot{W}}{n}$

$$T = \frac{5252(2500)}{2000} = 6565 \text{ lb ft} = 78{,}780 \text{ lb in.}$$

2. From Eq. (4.4), $\tau = \dfrac{16\,T}{\pi\,d^3}$

$$\tau = \frac{16\,(78{,}780)}{\pi\,(2)^3} = 50{,}153 \text{ psi} = 50 \text{ ksi} \qquad \blacksquare$$

3. For a hollow shaft,

$d_o^3 - (0.9)^4 d_o^3 = 2^3$

$0.344\ d_o^3 = 8$

$d_o = 2.86$ in. ■

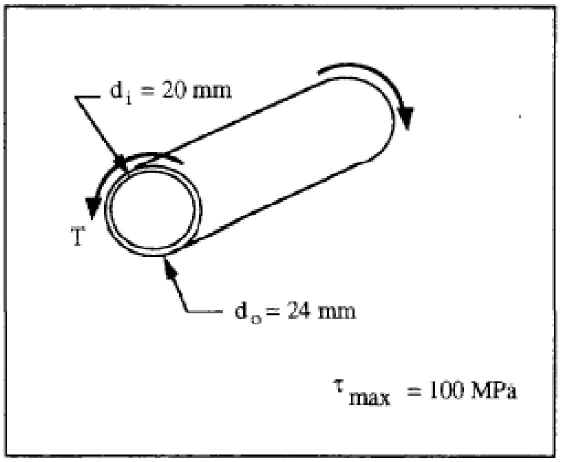
4. $$\frac{\text{Wt. hollow}}{\text{Wt. solid}} = \frac{\text{Area hollow}}{\text{Area solid}} = \frac{2.86^2 - (0.9 \times 2.86)^2}{2^2} = 0.39$$ ■

Comment: It is more economical to use a hollow shaft when pure shear stress is involved.

SOLUTION (4.10)
Known: The maximum shear stress is given for a hollow shaft of known geometry subjected to pure torsion.

Find: Determine the torque that produces the given maximum shear stress.

Schematic and Given Data:



Assumptions:
1. The shaft is straight.
2. The material is homogeneous and perfectly elastic.
3. There are no stress raisers.

Analysis:

1. From Eq. (4.3), $\tau = \dfrac{Tr}{J} = \dfrac{16Td_o}{\pi(d_o^4 - d_i^4)}$

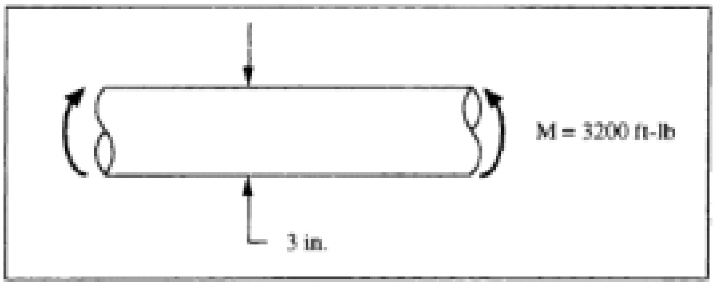
2. $T = \dfrac{\pi(100)(24^4 - 20^4)}{16(24)} = 0.141 \times 10^6 \text{ N·mm} = 141 \text{ N·m}$ ■

SOLUTION (4.15)

Known: A straight round shaft is subjected to a known bending moment.

Find: Calculate the nominal bending stress at the surface.

Schematic and Given Data:



Assumptions:

1. The bar is initially straight.
2. The material is homogeneous and perfectly elastic.
3. There are no stress raisers.

Analysis:

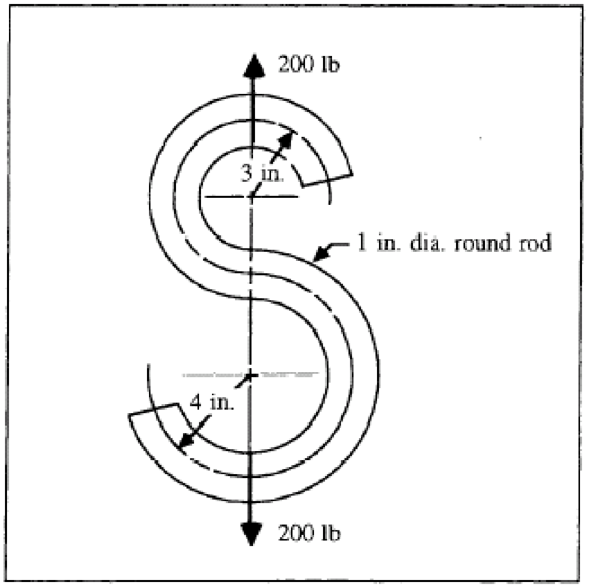
From Eq. (4.8), $\sigma = \dfrac{32M}{\pi d^3}$ for a round bar.

$$\sigma = \frac{32(38{,}400 \text{ in. lb})}{\pi(3 \text{ in.})^3} = 14{,}487 \text{ psi} = 14.5 \text{ ksi} \qquad \blacksquare$$

SOLUTION (4.19)
Known: A known force is exerted on an S-hook.

Find: Determine the location and magnitude of the maximum tensile stress.

Schematic and Given Data:



Assumption: Material is homogeneous and perfectly elastic.

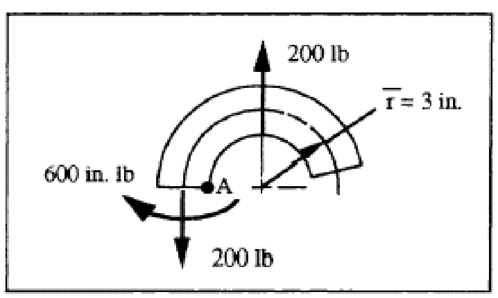
Analysis:

1. At point A, the tensile stress due to bending is

$$\sigma = \frac{32M}{\pi d^3} K_t \quad \text{[From Eq. (4.11)]}$$

The tensile stress due to tension is

$$\sigma = \frac{P}{A} = \frac{4P}{\pi d^2} \quad \text{[From Eq. (4.1)]}$$



Thus, the combined tensile stress is

$$\sigma = \frac{32M}{\pi d^3} K_t + \frac{4P}{\pi d^2}$$

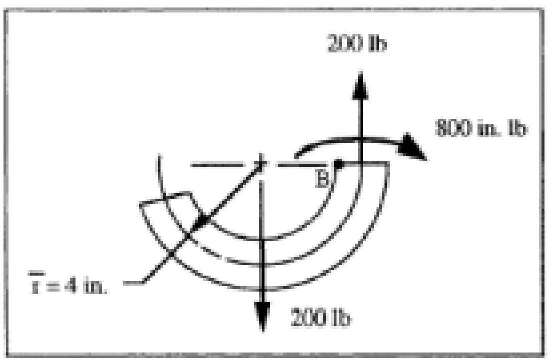
2. From Fig. 4.11, for $\frac{\bar{r}}{c} = \frac{3}{0.5} = 6$, $K_t = 1.14$.

3. $\sigma = (1.14)\,\frac{32(600)}{\pi(1)^3} + \frac{4(200)}{\pi(1)^2} = 6{,}967 + 255$
   $= 7{,}222$ psi
   At point A, $\sigma = 7.2$ ksi.
4. At point B, from Fig. 4.11, for

$$\frac{\bar{r}}{c} = \frac{4}{0.5} = 8,\ K_t = 1.10$$

$$\sigma = (1.10)\,\frac{32(800)}{\pi(1)^3} + \frac{4(200)}{\pi(1)^2}$$

$$= 8{,}964 + 255 = 9{,}219 \text{ psi.}$$



At point B, $\sigma = 9.2$ ksi. This point corresponds to the location of the maximum tensile stress. ■

Comment: The inner fiber is stressed more than the outer fiber because the stresses due to the direct tension and bending are of the same sign, and hence, add up to give a large resultant stress.

SOLUTION (4.34)

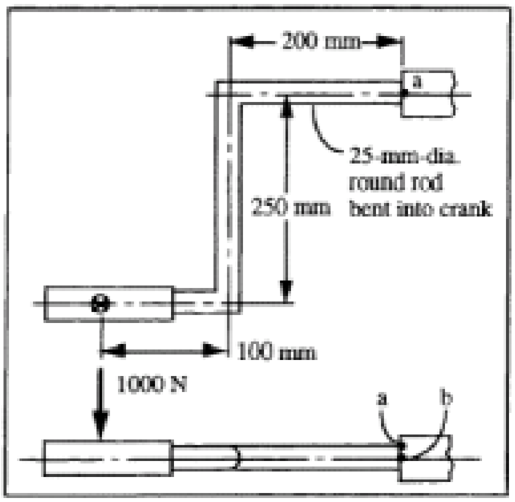
Known: A static vertical load is applied to the handle of a hand crank.

Find:

(a) Copy the drawing and mark on it the location at highest bending stress. Make a three-dimensional Mohr-circle representation of the stresses at this point.

(b) Mark on the drawing the location at highest combined torsional and transverse shear stress. Make a three-dimensional Mohr-circle representation of the stresses at this point.

Schematic and Given Data:



Assumptions:

1. The weight of the hand crank is negligible.
2. The effect of the stress concentration is negligible.
3. The crank material is homogeneous and perfectly elastic.

Analysis:

1. For bending, Eq. (4.8)

$$\sigma = \frac{32M}{\pi d^3} = \frac{32(300\text{ mm})(1000\text{ N})}{\pi(25\text{ mm})^3} = 195.6\text{ MPa}$$

2. For torsion, Eq. (4.4)

$$\tau = \frac{16T}{\pi d^3} = \frac{16(250\text{ mm})(1000\text{ N})}{\pi(25\text{ mm})^3} = 81.5\text{ MPa}$$

3. For transverse shear, Eq. (4.13)

$$\tau_{tr} = \frac{4V}{3A} = \frac{4(1000\text{ N})}{3\pi(12.5\text{ mm})^2} = 2.7\text{ MPa}$$

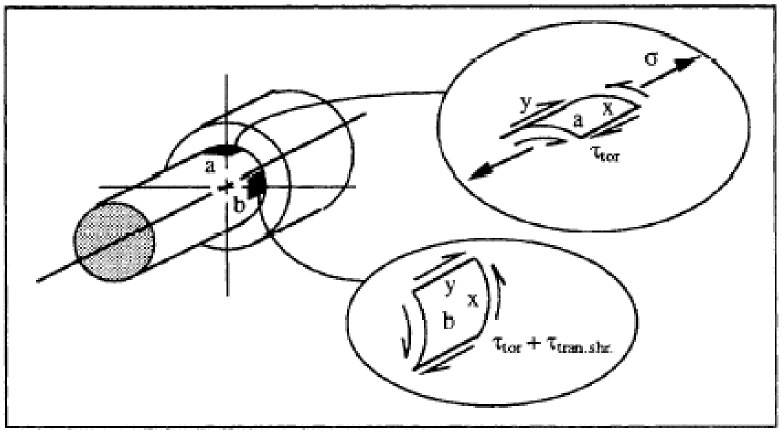
4. From the Mohr circle for "a",

$\tau_{max} = 128$ MPa, $\sigma_{max} = 225$ MPa.

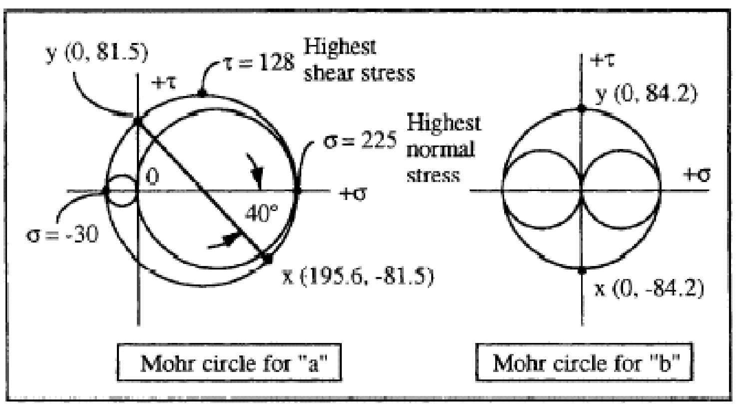
From the Mohr circle for "b",

$\tau_{max} = 84.2$ MPa, $\sigma_{max} = 84.2$ MPa.

5.



6.



SOLUTION (4.41)

Known: A small pressurized cylinder is attached at one end and loaded with a pipe wrench at the other. The stresses due to the internal pressure and the pipe wrench are known.

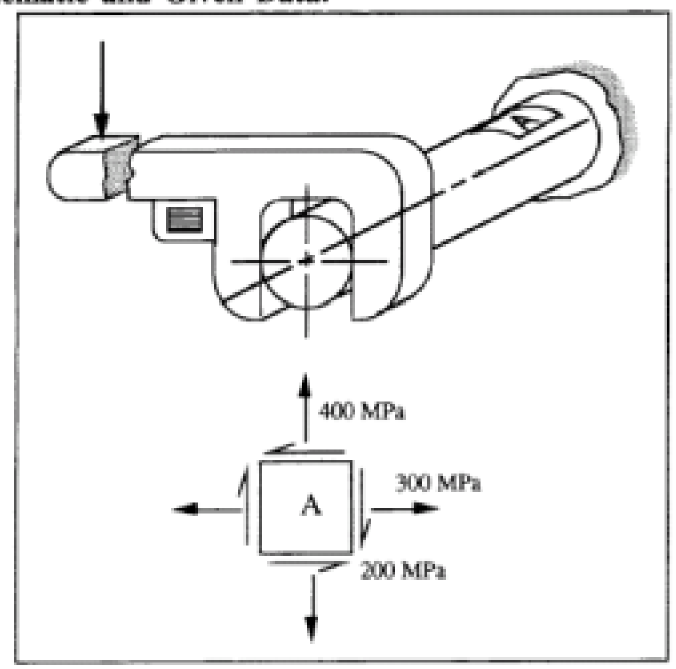
Find:

(a) Draw a Mohr-circle representation of the state of stress at point A.

(b) Determine the magnitude of the maximum shear stress at A.

(c) Sketch the orientation of a principal element, and show all stresses acting on it.

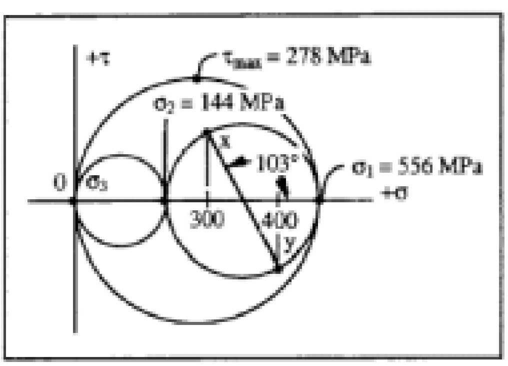
Schematic and Given Data:



Assumption: The "positive-clockwise" rule is used.

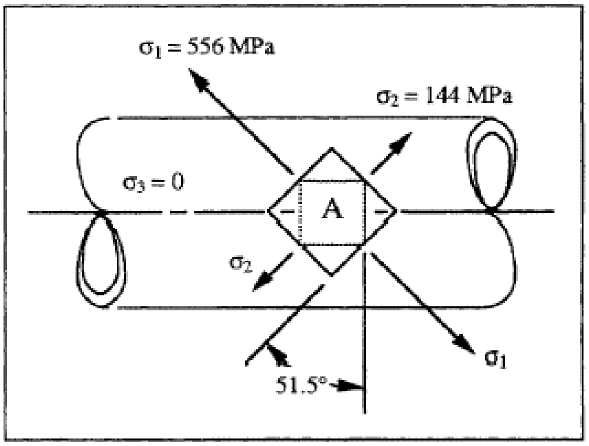
Analysis:

1.



2. The maximum shear stress at A is $\tau_{max} = 278$ MPa ■

3.

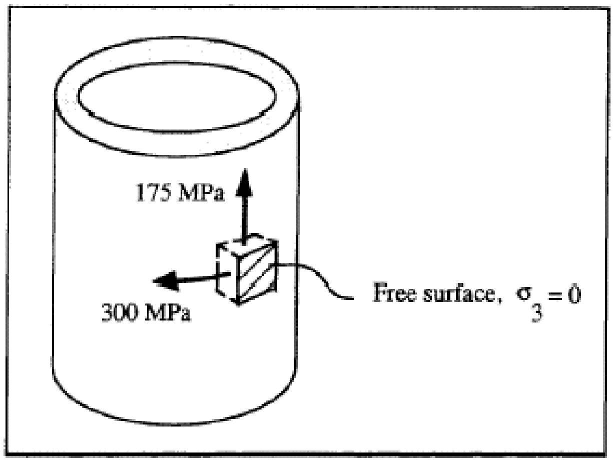


SOLUTION (4.48)

Known: A cylinder is internally pressurized to a known pressure which causes known tangential and axial stresses in the outer surface.

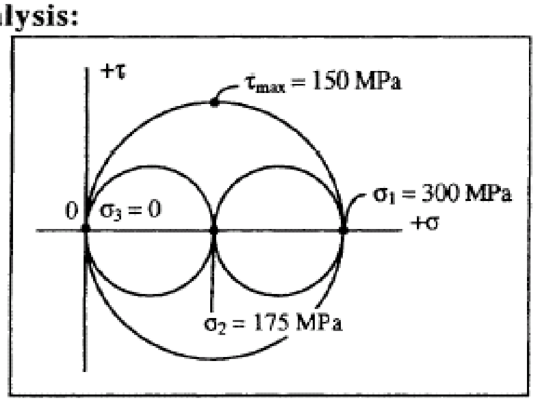
Find: Draw a Mohr-circle representation of the stresses in the outer surface and determine the maximum shear stress experienced.

Schematic and Given Data:



Assumption: The "positive-clockwise" rule is used.

Analysis:



The maximum shear stress in the outer surface is 150 MPa.

Comments:

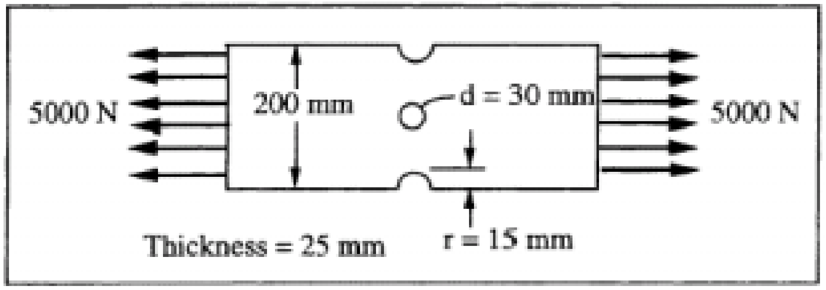
1. The principal stress σ_3 is zero at the outer surface.
2. The tangential stress is analytically twice the axial stress at the outer surface of an internally pressurized cylinder. We could speculate in this problem that a stress concentration existed which increased the axial stress from 150 MPa to 175 MPa.

SOLUTION (4.56)

Known: The geometry and the tensile force acting on a specimen are known.

Find: Determine the maximum stress at both the hole and the notch.

Schematic and Given Data:



Assumption: The material is homogeneous and elastic.

Analysis:

1. The nominal stress at the hole and notch is $\sigma_{nom} = \dfrac{5000}{(200 - 60)25} = 1.43$ MPa.
2. For the notch: H/h = 200/170 = 1.2, r/h = 15/170 = 0.09. From Fig (4.39b), K_t = 2.45.
   Therefore, the maximum stress in the notch $\sigma_{Nmax} = 2.45(1.43) = 3.5$ MPa.
3. For the hole: d/b = 30/200 = 0.15. From Fig (4.40b), $K_t = 2.5$.
   Therefore, the maximum stress in the hole $\sigma_{hmax} = 2.5(1.43) = 3.6$ MPa.

Comments:

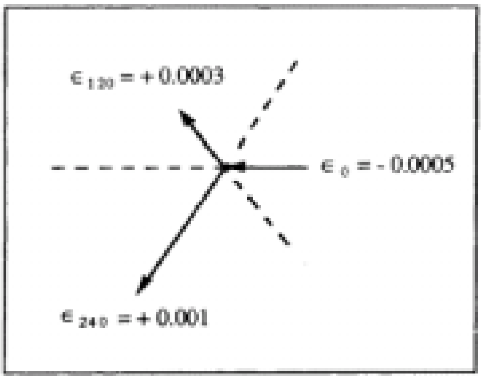
1. The above stress concentration factor is theoretical based on a theoretical elastic, homogeneous, isotropic material.
2. The value of the maximum stress at the hole is greater than that at the notch.
3. Divide the plate into two notched bars. For each notched bar, $\sigma_{nom} = 1.43$ MPa, H/h = 1.43, r/h = 0.21, $K_t = 1.95$, $\sigma_{max} = 2.8$ MPa.

SOLUTION (5.3)

Known: Three readings are obtained from an equiangular strain gage rosette mounted on a free and unloaded surface of a part.

Find: Determine the magnitude of the principal strains and their orientation with respect to the 0° gage. Check the results with a Mohr circle.

Schematic and Given Data:



Assumption: The three known strains are all linear strains.

Analysis:

1. From Eq. (5.1),

$$\varepsilon_{1,2} = \frac{\varepsilon_0+\varepsilon_{120}+\varepsilon_{240}}{3} \pm \sqrt{\frac{(2\varepsilon_0-\varepsilon_{120}-\varepsilon_{240})^2}{9} + \frac{(\varepsilon_{120}-\varepsilon_{240})^2}{3}}$$

$$\varepsilon_{1,2} = \frac{-0.0005+0.0003+0.001}{3} \pm \sqrt{\frac{(-0.001-0.0003-0.001)^2}{9} + \frac{(0.0003-0.001)^2}{3}}$$

$= 0.000267 \pm 0.000867$

Thus, $\varepsilon_1 = 0.00113$ m/m, $\varepsilon_2 = -0.000600$ m/m. ■

2. From Eq. (5.2),

$$\tan 2\alpha = \frac{\sqrt{3}(\varepsilon_{120}-\varepsilon_{240})}{2\varepsilon_0-\varepsilon_{120}-\varepsilon_{240}}$$

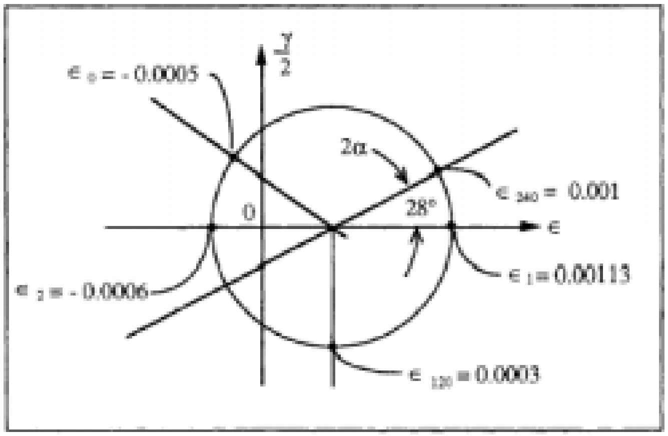
$$\tan 2\alpha = \frac{\sqrt{3}(0.0003-0.001)}{-0.001-0.0003-0.001} = 0.527$$

$$2\alpha = 28^{\circ},\ 208^{\circ}$$

So, $\alpha = 14°, 104°$

Use the rule that the higher principal strain always lies within 30° of the algebraically highest of ϵ_0, ϵ_{120}, ϵ_{240}. Then, ϵ_1 is 74° clockwise from the 0° gage and ϵ_2 is 164° clockwise from the 0° gage. ■

3.

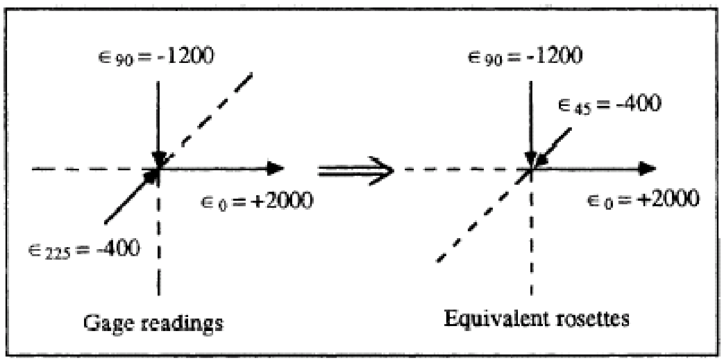


SOLUTION (5.8)

Known: Three readings are obtained from a rectangular strain gage rosette mounted on a free and unloaded surface.

Find: Determine the magnitude of the principal strains and their orientation with respect to the 0° gage. Check the results with a Mohr circle.

Schematic and Given Data:



Assumption: The three known strains are all linear strains.

Analysis:

1. From Eq. (5.3), $\varepsilon_{1,2} = \dfrac{\varepsilon_0 + \varepsilon_{90}}{2} \pm \sqrt{\dfrac{(\varepsilon_0 - \varepsilon_{45})^2 + (\varepsilon_{45} - \varepsilon_{90})^2}{2}}$

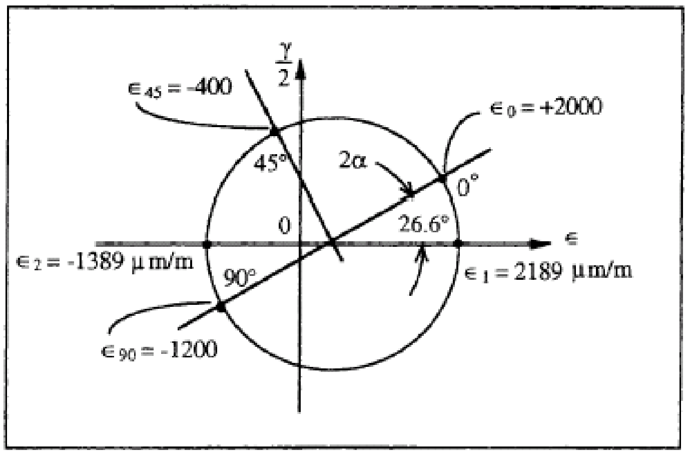
$$= \frac{(2000 - 1200)}{2} \pm \sqrt{\frac{2400^2 + 800^2}{2}} = 400 \pm 1789$$

Thus, $\varepsilon_1 = 2189$ μm/m and $\varepsilon_2 = -1389$ μm/m. ■

2. From Eq. (5.4), $\tan 2\alpha = \dfrac{\varepsilon_0 - 2\varepsilon_{45} + \varepsilon_{90}}{\varepsilon_0 - \varepsilon_{90}} = \dfrac{2000 + 800 - 1200}{3200} = 0.5$

So, $\alpha = 13.3°$. Discrimination between the two principal axes can be based on the rule that the algebraically greater principal strain makes an angle of less than 45° with the algebraically larger of strains ε_0 and ε_{90} . Thus, ε_1 is 13.3° clockwise from the +2000 μm/m gage. ■

3.

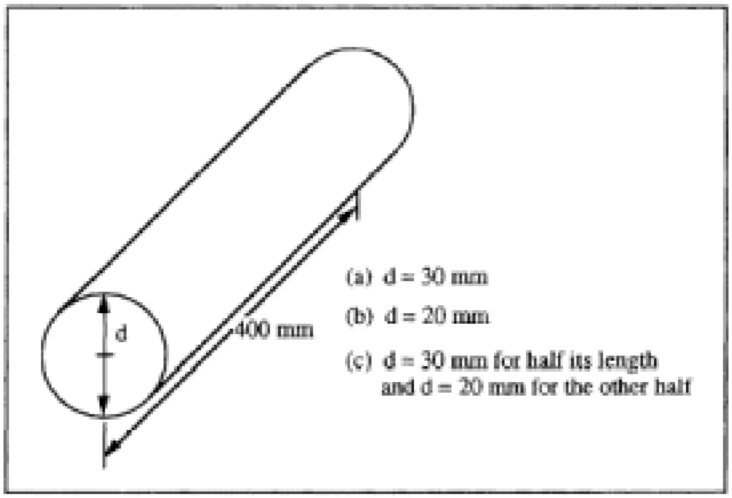


SOLUTION (5.13)

Known: Diameters are known for solid round steel shafts 400 mm long.

Find: Calculate the torsional spring constant in (N·m torque per degree of angular deflection).

Schematic and Given Data:



Assumption: The shaft deflection is elastic.

Analysis:

1. From Table 5.2,

$$\theta = \frac{TL}{JG} = \frac{32TL}{\pi d^4 G} \text{ [in rad] } = \frac{32TL}{\pi d^4 G}\left(\frac{180}{\pi}\right) = \frac{584TL}{d^4 G} \text{ [in degrees]}$$

2. From Table 5.1 (Case 2),
$K = \frac{T}{\theta} = \frac{d^4 G}{584L}$ where G = 79 GPa for steel (Appendix C-1)

3. For d = 30 mm,
$$K = \frac{0.030^4(79 \times 10^9)}{584(0.400)} = 274 \text{ N-m/deg.} \qquad ■$$

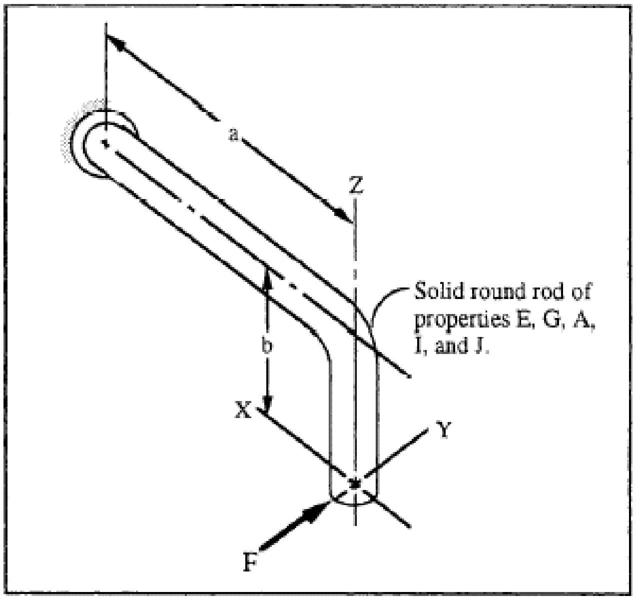
4. For d = 20 mm,
$$K = \frac{0.020^4(79 \times 10^9)}{584(0.400)} = 54 \text{ N-m/deg.} \qquad ■$$

5. For 200 mm lengths, K = 548 N·m/deg. and 108 N·m/deg. for 30 mm and 20 mm diameters respectively. Applying a torque of 548 N·m will cause a total deflection of $1 + \frac{548}{108} = 6.07$ deg. Therefore, $K = \frac{548}{6.07} = 90$ N·m/deg.

SOLUTION (5.17)
Known: A bracket is loaded with a force in the Y-direction.

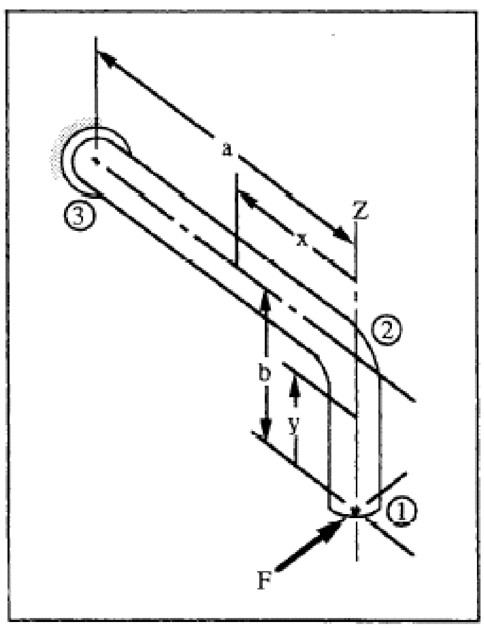
Find: Derive an expression for the deflection of the free end in the Y-direction.

Schematic and Given Data:



Assumptions:
1. The bracket remains elastic.
2. Neglect deflection due to shear force.

Analysis:
1.



2. In section 1-2, $M = Fy$ (neglect $V = F$).
   In section 2-3, $M = Fx$; $T = Fb$
3. Using Castigliano's Method, (Table 5.3),

$$U = \int_o^b \frac{(Fy)^2}{2EI}\,dy + \int_o^a \frac{(Fx)^2}{2EI}\,dx + \int_o^a \frac{(Fb)^2}{2GJ}\,dx$$

$$U = \frac{F^2}{2EI}\int_o^b y^2\,dy + \frac{F^2}{2EI}\int_o^a x^2\,dx + \frac{F^2b^2}{2GJ}\int_o^a dx$$

$$U = \frac{F^2b^3}{6EI} + \frac{F^2a^3}{6EI} + \frac{F^2b^2a}{2GJ}$$

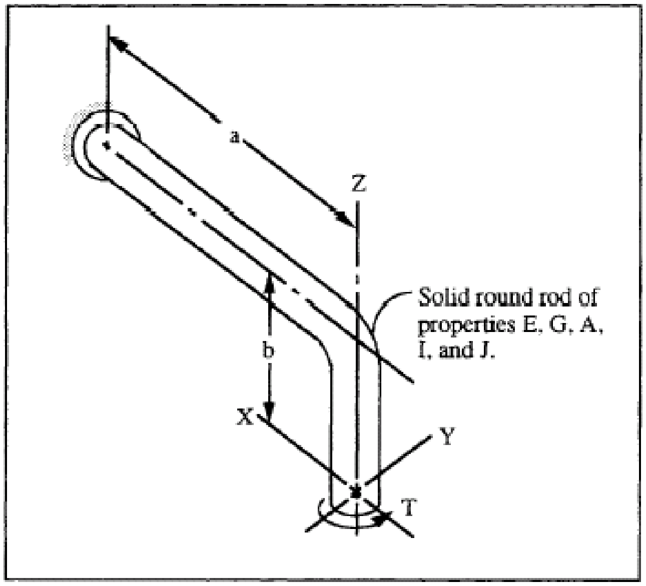
$$\delta = \frac{\partial U}{\partial F} = \frac{Fb^3}{3EI} + \frac{Fa^3}{3EI} + \frac{Fb^2a}{GJ}$$

■

SOLUTION (5.18)
Known: A bracket is loaded with a torque about the Z-axis.

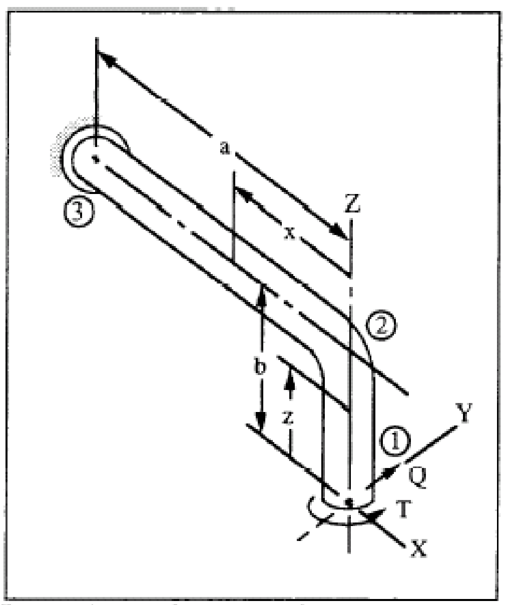
Find: Derive an expression for the resulting deflection of the free end in the Y-direction.

Schematic and Given Data:



Assumptions:
1. The bracket remains elastic.
2. The deflection due to shear force is negligible.

Analysis:
1.



2. In section 1-2, $M_{12} = Qz$, $T_{12} = T$
   where Q is a dummy load.
   In section 2-3, $M_{23} = Qx + T$, $T_{23} = Qb$

3. Using Castigliano's Method (Table 5.3),

$$\delta = \int_o^b \frac{M_{12}(\partial M_{12}/\partial Q)}{EI}\,dz + \int_o^b \frac{T_{12}(\partial T_{12}/\partial Q)}{GJ}\,dz$$

$$+ \int_o^a \frac{M_{23}(\partial M_{23}/\partial Q)}{EI}\,dx + \int_o^a \frac{T_{23}(\partial T_{23}/\partial Q)}{GJ}\,dx$$

where shear force V = Q is neglected.

$$\delta = \int_o^b \frac{Qz(z)}{EI}\,dz + \int_o^b \frac{T(0)}{GJ}\,dz + \int_o^a \frac{(Qx + T)(x)}{EI}\,dx + \int_o^a \frac{Qb(b)}{GJ}\,dx$$

4. Set Q = 0, then

$$\delta = 0 + 0 + \int_o^a \frac{Tx}{EI}\,dx + 0$$

$$\delta = \frac{Ta^2}{2EI}$$

■

SOLUTION (5.23)

Known: In order to reduce the deflection of the I-beam cantilever, a support is to be added at S.

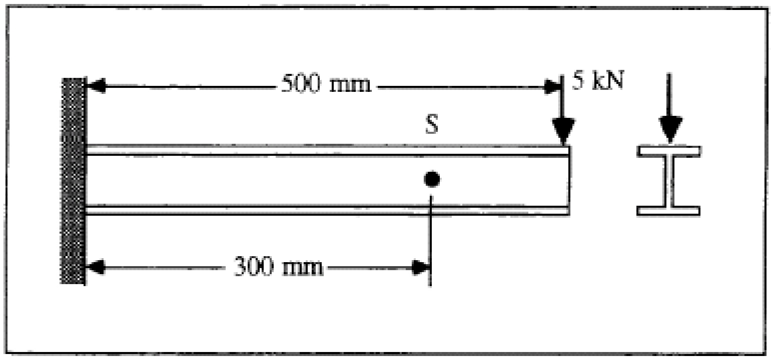
Find:

(a) Determine the magnitude of the vertical force at S needed to reduce the deflection at that point to zero.

(b) Determine the force needed to cause an upward deflection at S of 5 mm.

(c) Comment about the effect of these forces at S on the bending stresses at the point of beam attachment.

Schematic and Given Data:



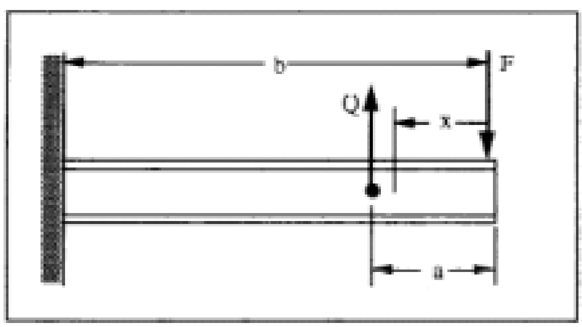
Assumptions:

1. The beam remains elastic.
2. Transverse shear deflection is negligible.

Analysis:

(a)

1.



Neglect transverse shear, for $0 < x < a$, $M_{oa} = Fx$; and for $a < x < b$, $M_{ab} = Fx - Q(x - a)$.

2. Using the general energy equation in Table 5.3:

$$U = \int_0^a \frac{M_{oa}^2}{2EI}\,dx + \int_a^b \frac{M_{ab}^2}{2EI}\,dx$$

$$U = \int_0^a \frac{F^2x^2}{2EI}\,dx + \int_a^b \frac{(F^2 - 2FQ + Q^2)\,x^2 + (2FQa - 2Q^2a)\,x + Q^2a^2}{2EI}\,dx$$

$$U = \frac{F^2}{2EI}\,\frac{a^3}{3} + \frac{F^2 - 2FQ + Q^2}{2EI}\left(\frac{b^3 - a^3}{3}\right) + \frac{2FQa - 2Q^2a}{2EI}\left(\frac{b^2 - a^2}{2}\right) + \frac{Q^2a^2}{2EI}\,(b-a)$$

3. For zero deflection where Q is applied:

$$\frac{\partial U}{\partial Q} = 0 = \left(\frac{-2F + 2Q}{2EI}\right)\left(\frac{b^3 - a^3}{a^3}\right) + \frac{2Fa - 4Qa}{2EI}\left(\frac{b^2 - a^2}{2}\right) + \frac{2Qa^2}{2EI}\,(b - a)$$

$$Q\left[\frac{b^3 - a^3}{3} - a(b^2 - a^2) + a^2(b - a)\right] = F\left[\frac{b^3 - a^3}{3} - \frac{a(b^2 - a^2)}{2}\right]$$

$$Q = F\left[\frac{\dfrac{b^3 - a^3}{3} - \dfrac{a(b^2 - a^2)}{2}}{\dfrac{b^3 - a^3}{3} - a(b^2 - a^2) + a^2(b-a)}\right]$$

4. Substituting (letting 600 mm = 1 arbitrary unit of length) b = 1, a = 0.4. Then,

$$Q = F\left[\frac{\dfrac{1 - 0.064}{3} - \dfrac{0.4\,(1 - .016)}{2}}{\dfrac{1 - 0.064}{3} - 0.4(1 - 0.16) + 0.16(0.6)}\right]$$

$$Q = F\left[\frac{0.312 - 0.168}{0.312 - 0.336 + 0.096}\right] = \frac{0.144}{0.072}\,F$$

Q = 2F: but F = 5 kN, therefore Q = 10 kN. ■

(b) For deflections, δ, other than zero where Q is applied:

$$\frac{\partial U}{\partial Q} = \delta = \frac{1}{EI}\left\{F\left[-\frac{b^3 - a^3}{3} + a\left(\frac{b^2 - a^2}{2}\right)\right]\right\} + \frac{1}{EI}\left\{Q\left[\frac{b^3 - a^3}{3} - a(b^2 - a^2) + a^2\,(b-a)\right]\right\}$$

$$Q = \frac{EI\delta + F\left[\dfrac{b^3 - a^3}{3} - a\left(\dfrac{b^2 - a^2}{2}\right)\right]}{\dfrac{b^3 - a^3}{3} - a(b^2 - a^2) + a^2(b - a)}$$

Using base units of m, N, Pa:

$$Q = \frac{EI\delta + 5000\left[\dfrac{0.5^3 - 0.2^3}{3} - 0.2\left(\dfrac{0.5^2 - 0.2^2}{2}\right)\right]}{\dfrac{0.5^3 - 0.2^3}{3} - 0.2(0.5^2 - 0.2^2) + 0.2^2(0.3)}$$

$$Q = \frac{EI\delta + 5000\,[0.039 - 0.021]}{0.039 - 0.042 + 0.012} = \frac{EI\delta + 90}{0.009}$$

$$Q = \frac{EI\delta}{0.009} + 10{,}000; \text{ for } \delta = .005 \text{ m}, \ Q = \frac{5EI}{9} + 10{,}000$$ ■

Note that this solution is valid for any value of δ including zero. For $\delta = 0$, Q = 10,000 N, as determined in part a. Note that for the special case of $\delta = 0$, the values of "E" and "I" are not relevant (provided they are such that the beam deflections are not sufficient to significantly alter the moment equations).

(c) In (a), the moment at the support is changed from 5 kN(0.5 m) = 2.5 kN•m to 5(0.5 m) - 10(0.3) = -0.5 kN•m. Hence, stresses are reduced by 80% and reversed in direction. This is true regardless of values of "E" and "I".

In (b), the effect of Q would be to reverse the stress and give a magnitude exceeding 20% of the Q = 0 stress. The greater the product EI, the greater the stress. ■

SOLUTION (5.29)

Known: A boom and tie-rod arrangement is supporting a load of 6 kN. The tie-rod is made of steel having a tensile yield strength of 400 MPa.

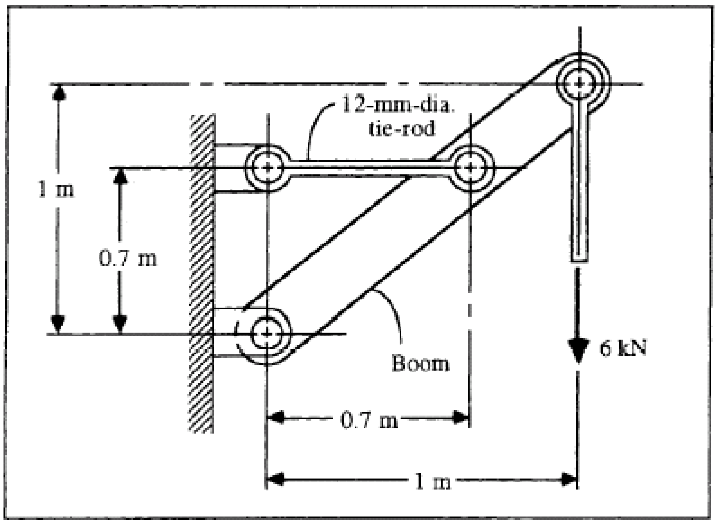
Find:

(a) Determine the safety factor of the tie-rod with respect to static yielding.

(b) Determine the safety factor of the tie-rod if the vertical rod is rotated 180° so that the 6 kN load acts upward.

(c) Draw a conclusion with respect to the relative desirability of designing machines with column members loaded in tension vs. loaded in compression.

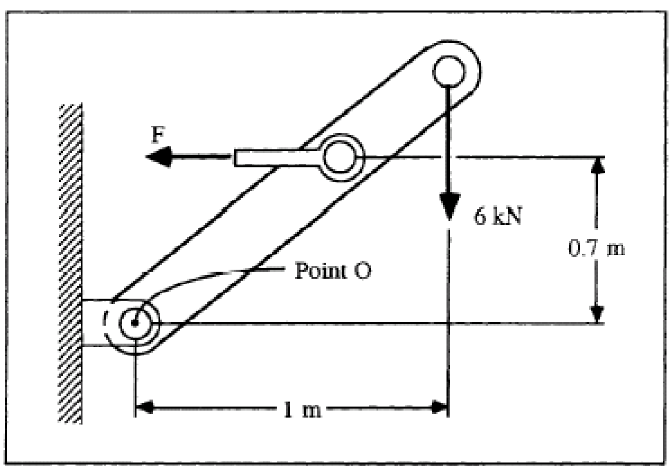
Schematic and Given Data:



Assumption: The tie-rod is straight.

Analysis:

(a)



$$\Sigma M_o = 0: \quad F = \frac{6\text{ kN}(1\text{ m})}{0.7\text{ m}} = 8.57\text{ kN}$$

Tensile stress in the tie rod is $\sigma = \frac{P}{A} = \frac{8570}{36\,\pi\text{ mm}^2} = 75.8\text{ MPa}$

$$SF = \frac{400}{75.8} = 5.3$$

■

(b) $\sigma = 75.8$ MPa in compression

From Appendix B-1, $\rho = \frac{d}{4} = 3$ mm, $L = L_e = 700$ mm $\Rightarrow \frac{L_e}{\rho} = 233.3$

Euler-Johnson tangent point [Eq. (5.13)] is at

$$\frac{L_e}{\rho} = \left(\frac{2\pi^2 E}{S_y}\right)^{\frac{1}{2}} = \left[\frac{2\pi^2(207 \times 10^3)}{400}\right]^{\frac{1}{2}} = 101$$

where $E = 207 \times 10^9$ Pa (Appendix C-1).

Hence, Euler Eq. (5.11) applies:

$$S_{cr} = \frac{\pi^2 E}{(L_e/\rho)^2} = \frac{\pi^2(207 \times 10^3)}{(233.3)^2} = 37.5\text{ MPa}$$

Since 75.8 > 37.5, the rod will fail in buckling. ■

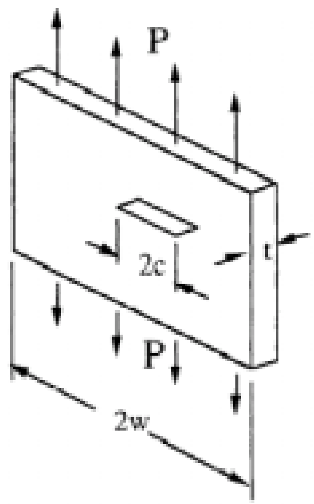
(c) Long, slender rods in compression can carry only a small fraction of the load they can carry in tension.

SOLUTION (6.7)

Known: A thin plate of known material is loaded in tension and has a central crack of given length perpendicular to the direction of the applied load.

Find: Estimate the highest tensile load that the plate will support.

Schematic and Given Data:



Leaded beryllium copper

$S_u = 98$ ksi

$S_y = 117$ ksi

$K_{Ic} = 70$ ksi in$^{0.5}$

$2w = 8$ in.
$t = 0.05$ in.
$2c = 1.5$ in.

$P = ?$

Assumptions:

1. The crack length is a small fraction of the plate width.
2. The tensile stress based on the net area (minus the area of the crack) is less than the yield strength.
3. Yielding has occurred within one small volume of the material at the crack root.
4. Crack propagation to total fracture occurs instantaneously when the limiting value of the stress intensity factor K_I equals or exceeds the fracture toughness K_{IC} for the material.

Analysis:

1. From Eq. (6.2), $\sigma_g = \dfrac{K_{Ic}}{1.8\sqrt{c}} = \dfrac{70}{1.8\sqrt{0.75}} = 44.9$ ksi

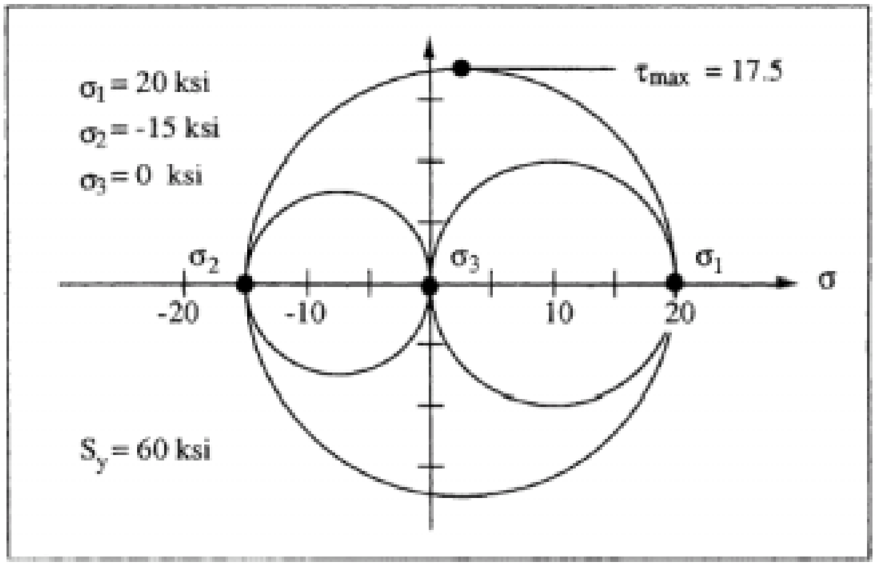
2. Since the area equals 2wt, $P = \sigma_g(2wt) = 44{,}900(8)(0.05) = 17{,}960$ lb ■

Comment: The P/A stress based on the net area, $t(2w - 2c)$, is 55.26 ksi which is less than $S_y = 117$ ksi. Hence the second assumption is satisfied.

SOLUTION (6.15)
Known: A machine component with given critical stresses is ductile, with yield strengths in tension and compression of 60 ksi.

Find: Determine the safety factor according to:
(a) the maximum-normal-stress theory
(b) the maximum-shear-stress theory
(c) the maximum-distortion-energy theory

Schematic and Given Data:



Assumption: The material is homogeneous.

Analysis:
1. From the above Mohr-circle, $\tau_{max} = (20 + 15)/2 = 17.5$ ksi

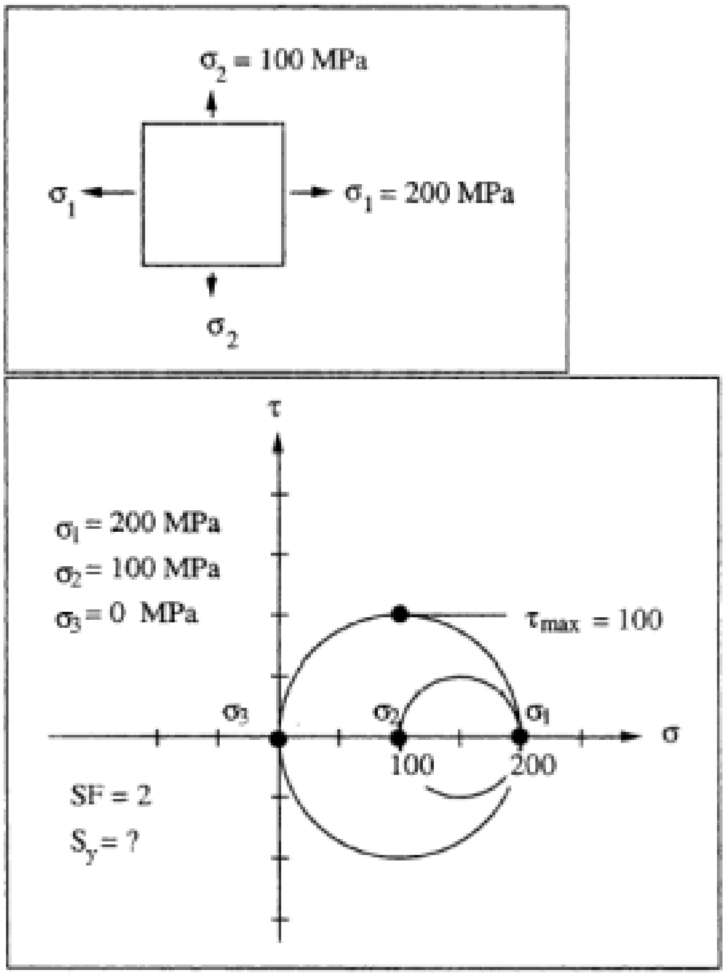
2. (a) For the maximum-normal-stress theory:
   SF = 60/20 = 3.0 ■
   (b) For the maximum-shear-stress theory:
   SF = $30/\tau_{max}$ = 30/17.5 = 1.72 ■
   (c) For the maximum-distortion-energy theory:
   SF = 60/S', where from Eq. (6.6)
   $S' = [\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2]^{1/2}$
   $= [(20)^2 - (20)(-15) + (-15)^2]^{1/2} = 30.5$
   thus, SF = 60/30.5 = 1.97 ■
3. The existence of a yield strength implies a ductile material for which:
   • maximum-distortion-energy theory is best
   • maximum-shear-stress theory may be acceptable
   • maximum-normal-stress theory is not appropriate

SOLUTION (6.21)

Known: The surface of a steel machine member is subjected to known principal stresses.

Find: Determine the tensile yield strength required to provide a safety factor of 2 with respect to initial yielding.

Schematic and Given Data:



Assumption: The material is homogeneous

Analysis:

1. Maximum-shear-stress theory:
   For $\sigma_1 = 200$ MPa, $\sigma_2 = 100$ MPa, $\sigma_3 = 0$
   $\tau_{max} = (0 + 200)/2 = 100$ MPa.
   Thus, for SF = 2, $S_y = 400$ MPa is required. ■
2. Maximum-distortion-energy theory:
   From Eq. (6.6),

$$\sigma_e = (\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2)^{1/2}$$
$$= [200^2 + 100^2 - (200)(100)]^{1/2} = 173.2 \text{ MPa.}$$

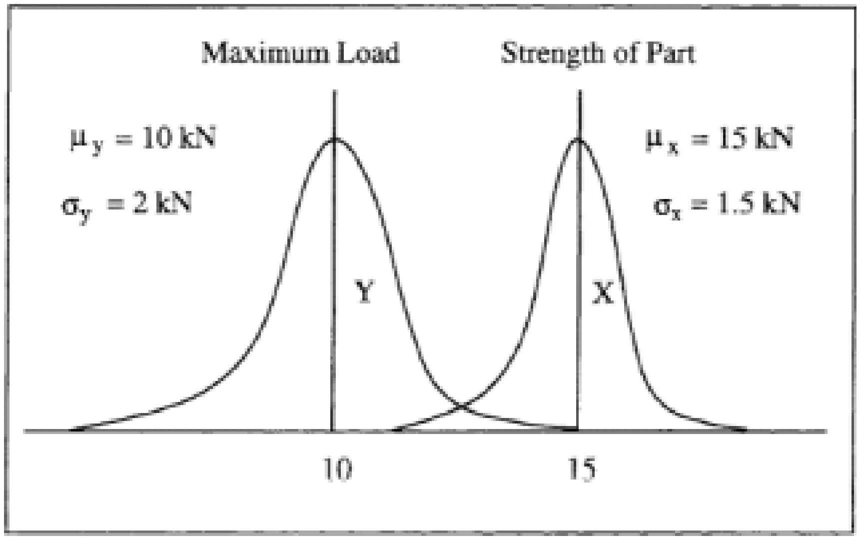
Thus, for SF = 2, $S_y = 346.4$ MPa is required. ■

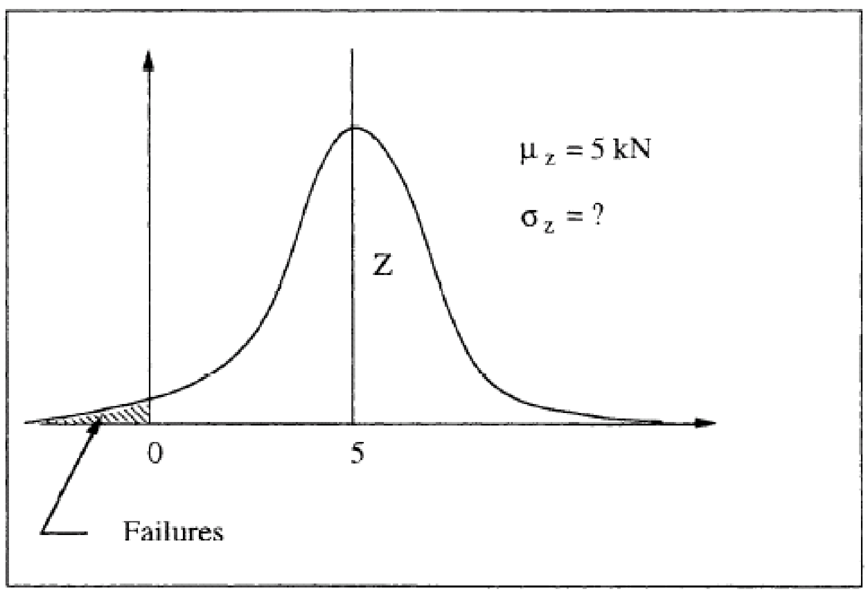
SOLUTION (6.34)

Known: The distribution of the maximum load encountered and the distribution of the part strength are known.

Find: Determine the failure percentage expected in service.

Schematic and Given Data:





Assumption: Both the part loads and the part strengths are normally distributed.

Analysis:

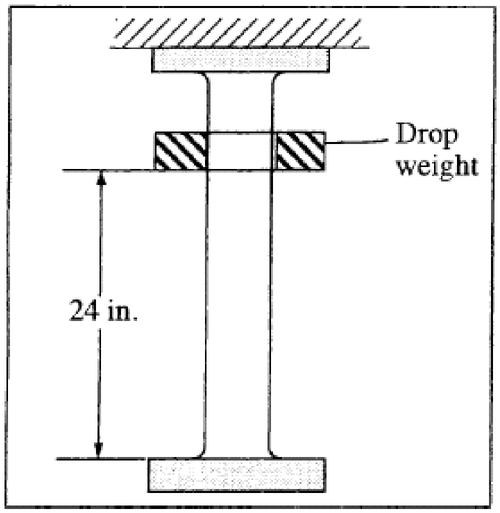
1. From Eq. (6.14), $\mu_z = \mu_x - \mu_y = 15 - 10 = 5$ kN
2. From Eq. (6.15), $\sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2} = \sqrt{1.5^2 + 2^2} = 2.5$ kN
3. Thus, failures begin at $\mu_z/\sigma_z = 5$ kN/2.5 kN = 2 standard deviations below the mean in the z distribution.
4. Figure 6.20 shows 2.27% failures. ■

SOLUTION (7.4)

Known: A vertical rod is subjected to an axial impact by a 100 lb weight dropped from a height of 2 ft. The rod is made of steel, with $S_y = 45$ ksi and $E = 30 \times 10^6$ psi.

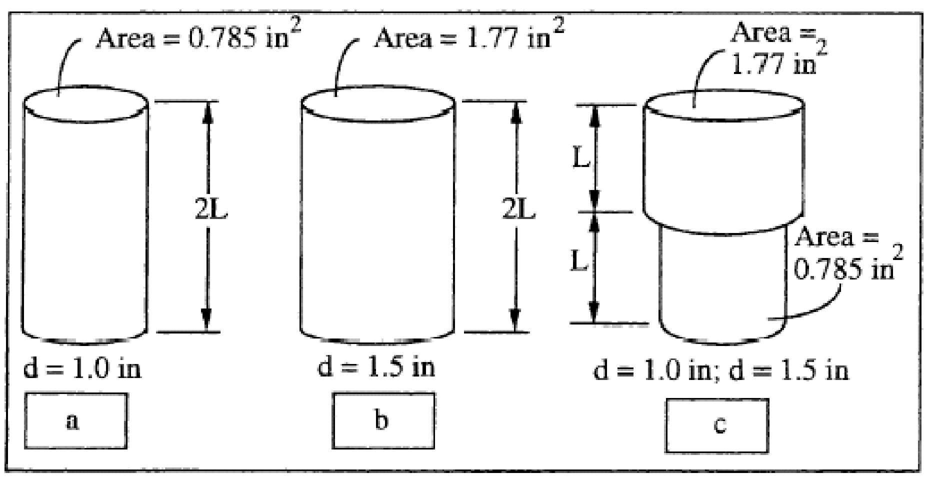
Find: The length of the member to avoid yielding for a diameter of (a) 1 in., (b) 1.5 in., and (c) 1 in. for half of its length, and 1.5 in for the other half.

Schematic and Given Data:



Assumptions:
1. The mass of the members are negligible.
2. Neglect any stress concentrations.

Analysis: Neglecting stress concentration, the load capacities of the old and the new bars are the same.



(a) Diameter is 1.0 inches

1. For a rod with uniform axial stress, $\sigma = \sqrt{\frac{2UE}{V}}$, or

$$V = \frac{2UE}{\sigma^2} = \frac{(2)(100)(24)(30 \times 10^6)}{45{,}000^2} = 71 \text{ in.}^{3*}$$

2. $L = \frac{V}{A} = \frac{71}{\pi/4} = 90.5$ in. ■

(b) Diameter is 1.5 inches

1. $V = 71$ in.^{3}

2. $L = \frac{(71)(4)}{(\pi)(1.5)^2} = 40$ in. ■

(c) Diameter of 1.0 inches and 1.5 inches

1. Let V_S = small end volume. (Large end volume = $(1.5)^2 V_S$)
2. Small end stress = S_y = 45,000 psi

3. Large end stress = $\frac{45{,}000}{(1.5)^2}$

4. Small end energy = $U_S = \frac{V_S\sigma_S^2}{2E} = \frac{V_S(45{,}000)^2}{(2)(30 \times 10^6)}$

5. Large end energy = $U_L = \frac{V_L\sigma_L^2}{2E} = \frac{(2.25V_S)\left(\frac{45{,}000}{2.25}\right)^2}{(2)(30 \times 10^6)}$

6. $U = U_S + U_L = 2400$ in. lb. $= \frac{V_S(45{,}000)^2}{(2)(30 \times 10^6)}\left(1 + \frac{1}{2.25}\right)$

7. Therefore, $V_S = 49.1$ in.^{3}

8. $L_S = \frac{49.1}{\pi/4} = 62.6$ in.

9. Therefore, L = 125.2 in. ■

Comment: Note physically why, if the "excess diameter" were machined off, the rod could be shortened from 125 inches to 90 inches.

---

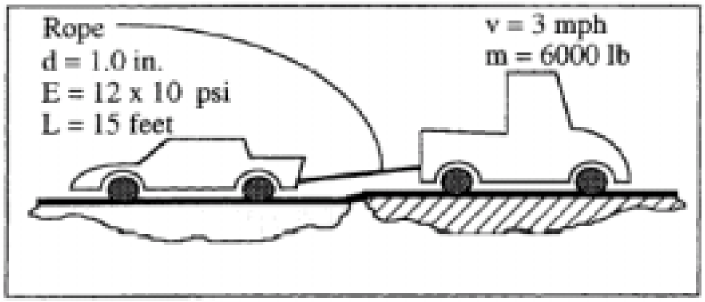
* This assumes a deflection which is negligible in comparison with the 24 in. drop.

SOLUTION (7.7)

Known: A 6000 lb tow truck, attempted to jerk a wrecked vehicle back onto a road using a 15 foot length of steel tow cable 1 in. in diameter ($E = 12 \times 10^6$ psi of the cable). The tow truck reached a speed of 3 mph at the instant the cable became taut, but the wrecked vehicle does not move. The cable breaks in the middle and the two 7.5 foot halves are connected in parallel for a second attempt.

Find: (a) Estimate the maximum impact force applied to the wrecked car and the stress produced in the cable. (b) Estimate the impact force and the cable stress developed in the second attempt if the wrecked vehicle still remains fixed.

Schematic and Given Data:



Assumptions:
1. The cable is attached rigidly to the masses of the cars.
2. Ignore the mass of the rope.
3. Neglect any stress concentrations.
4. Ignore damping due to internal friction within the rope.
5. The rope responds to the impact elastically.

Analysis:

(a) L = 15 feet

1. From Eq. (7.4a), $F_e = W\sqrt{v^2/g\delta_{st}}$ where v = 3 mph = 52.8 in./sec

2. $$\delta_{st} = \frac{PL}{AE} = \frac{(6000)(180)}{(.785)(12 \times 10^6)} = 0.1145 \text{ in.}$$

3. $$F_e = (6000)\sqrt{\frac{(52.8)^2}{(386)(0.1145)}} = (6000)(7.94) = 47{,}600 \text{ lb}$$ ■

4. $$\sigma = \frac{F_e}{A} = \frac{47{,}000 \text{ lb}}{0.785 \text{ in.}^2} = 60.6 \text{ ksi}$$ ■

(b) L = 7.5 feet (two parallel 1 in. cables)

1. δ_{st} is decreased by 4, therefore F_e is increased by 2. Thus, the impact force is 95,200 lb. ■
2. Both F_e and A are doubled; hence, σ_e remains at 60.6 ksi. ■

Comment: Note that in (a) and (b), the same volume of equally stressed material absorbs the same amount of energy.

SOLUTION (7.11)

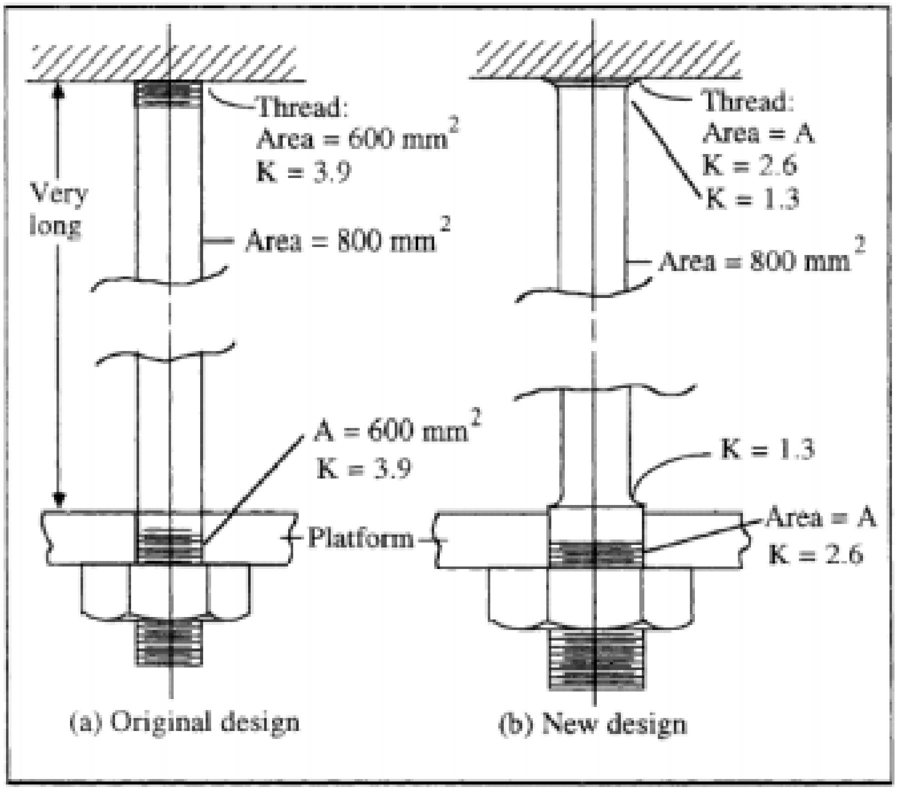
Known: A platform is suspended by long steel rods. The steel rod geometry is modified to obtain greater energy absorbing capacity.

Find:

(a) Determine the smallest effective threaded section area A that would provide maximum energy absorbing capacity for a new design.

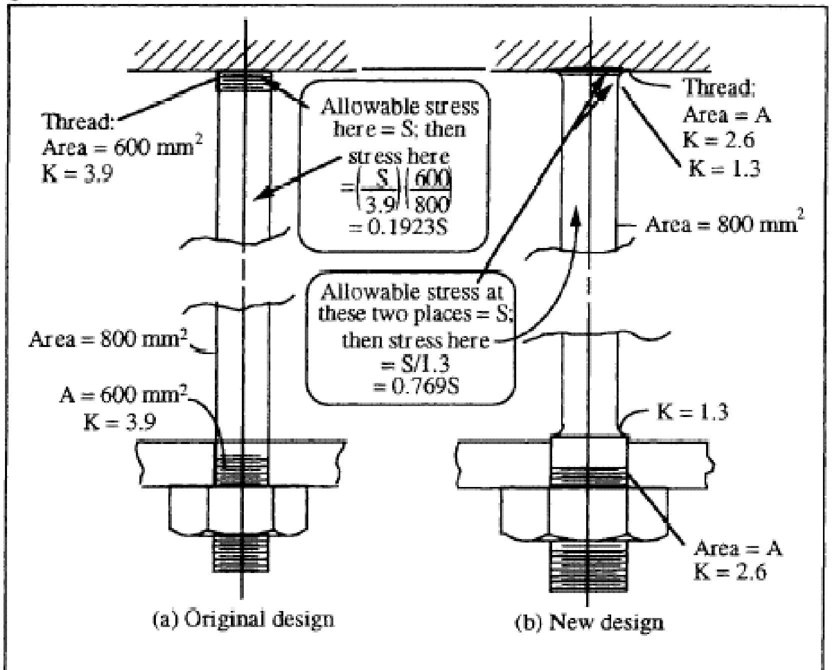
(b) Using this value of A, determine the increase in energy absorbing capacity that would be provided by the new rod design.

Schematic and Given Data:



Assumption: Under impact loading, the rod material exhibits brittle behavior.

Analysis:



1. To balance stress at thread and fillet, $A = 800\left(\frac{2.6}{1.3}\right) = 1600\ \text{mm}^2$ ■

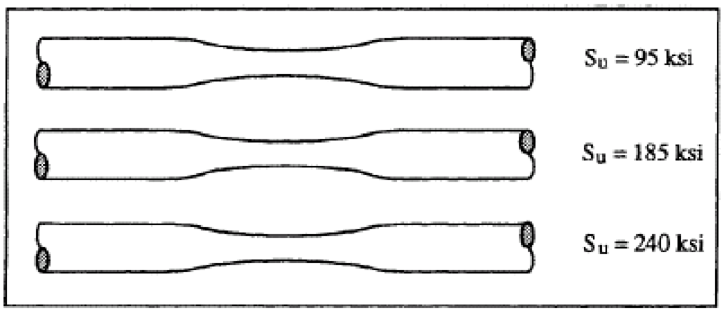
2. $\frac{U_b}{U_a} = \frac{\sigma_b^2}{\sigma_a^2} = \frac{0.7692^2}{0.1923^2} = 16$: An increase of 1500% (the new design has 16 times the capacity of the original). ■

SOLUTION (8.5)

Known: Standard R.R. Moore test specimens are made of steels having known ultimate tensile strengths.

Find: Estimate the rotating bending endurance limit and also the 10^3 cycle fatigue strength.

Schematic and Given Data:



Assumptions:

1. For steel, the tensile strength in psi is 500 times the Brinell hardness.
2. The curve in Fig. 8.5 is an accurate representation of the S-N data for steel.
3. For steel, the endurance limit in psi is 250 times the Brinell hardness.
4. For steel, the endurance limit for 10^3 cycle is 90% of the ultimate strength.

Analysis:

1. $S_n' = 0.5S_u$ in ksi.
2. S for 10^3 cycle $= 0.9S_u$
3.

S_u (ksi)	S_n (ksi)	S for 10^3 cycle (ksi)
95	47.5	85.5
185	92.5	166.5
240	100~125	216

Comments:

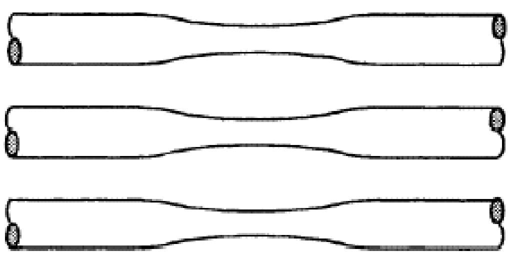
1. The relationship $S_n' = 0.5S_u$ is accurate only to ultimate tensile strength values of 200 ksi. The endurance limit may or may not continue to increase for greater tensile strength values depending on the composition of the steel.
2. For 10^3-cycle fatigue strength, actual stress is not as high as calculated values because of significant yielding.

SOLUTION (8.15)

Known: Standard R.R. Moore test specimens are made of steels having known ultimate tensile strengths.

Find: Estimate the endurance limit and also the 10^3 cycle fatigue strength for reversed torsional loading.

Schematic and Given Data:



Assumption: Figs. 8.5 and 8.11 can be used to estimate endurance limit and 10^3 cycle fatigue strength for reversed torsional loading.

Analysis:

1. $S_n' = 0.5\ S_u$ in ksi
2. $S_n = 0.58\ S_n'$
3. S for 10^3 cycle = 0.9 S_{us} where $S_{us} = 0.8\ S_u$ for steel

4.

S_u (ksi)	S_n (ksi)	S for 10^3 cycle (ksi)
95	47.5(0.58) = 27.55	68.4
185	92.5(0.58) = 53.65	133.2
240	(100~120)(0.58) = (58~69.6)	172.8

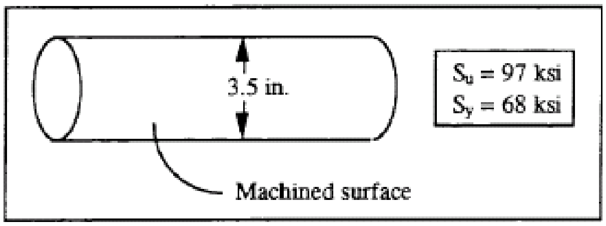
Comment: The relationship $S_n' = 0.5S_u$ is accurate only to Brinell hardness values of about 400 (200 ksi tensile strength).

SOLUTION (8.17)

Known: A steel bar having known S_u and S_y has average machined surfaces.

Find: Plot on log-log coordinates estimated S-N curves for (a) bending, (b) axial, and (c) torsional loading. For each of the three types of loading, determine the fatigue strength corresponding to (1) 10^6 or more cycles and (2) 5×10^4 cycles.

Schematic and Given Data:



Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_s.
4. The gradient factor, $C_G = 0.9$, for axial and torsional loading.

Analysis:

1. <u>Endurance limits</u>: (10^6 cycle strength)

   $S_n = S_n' C_L C_G C_s$

   For bending,

   $S_n' = 0.5\ S_u = 0.5(97) = 48.5$ ksi (Fig. 8.5)
   $C_L = 1$ (Table 8.1)
   $C_G = 0.9$ (Table 8.1)
   $C_s = 0.76$ (Fig. 8.13)
   $S_n = (48.5)(1)(0.9)(0.76) = 33.2$ ksi ■

   For axial,

   $S_n' = 48.5$ ksi
   $C_L = 1$
   $C_G = 0.8$ (between 0.7 and 0.9)
   $C_s = 0.76$
   $S_n = 48.5(1)(0.8)(0.76) = 29.5$ ksi ■

   For torsion,

   $S_n' = 48.5$ ksi
   $C_L = 0.58$
   $C_G = 0.9$
   $C_s = 0.76$
   $S_n = 48.5(0.58)(0.9)(0.76) = 19.2$ ksi ■

2. <u>10^3 cycle strength</u>

   For bending,

   $0.9S_u = 0.9(97) = 87.3$ ksi (Table 8.1)

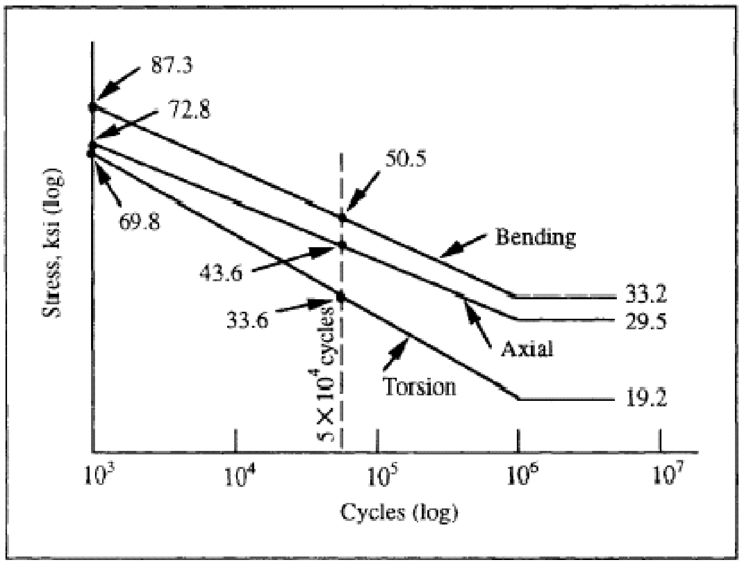
   For axial,

   $0.75S_u = 0.75(97) = 72.8$ ksi

   For torsion,

   $0.9S_{us} = 0.9(0.8)(97) = 69.8$ ksi

3. S-N curves



4. 5×10^4 cycle strength

Bending: 50.5 ksi ■

Axial: 43.6 ksi ■

Torsion: 33.6 ksi ■

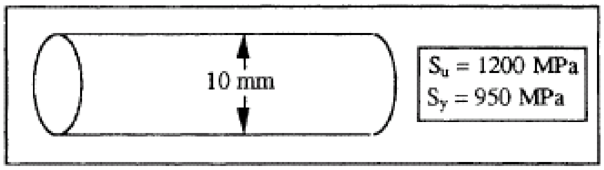
Comments:

1. The surface factor, C_S is not used for correcting the 10^3-cycle strength because for ductile parts the 10^3 strength which is close to the static strength, is unaffected by surface finish.
2. For critical designs, pertinent test data should be used rather than the preceding rough approximation.

SOLUTION (8.19)
Known: A steel bar having known S_u and S_y has a fine ground surface.

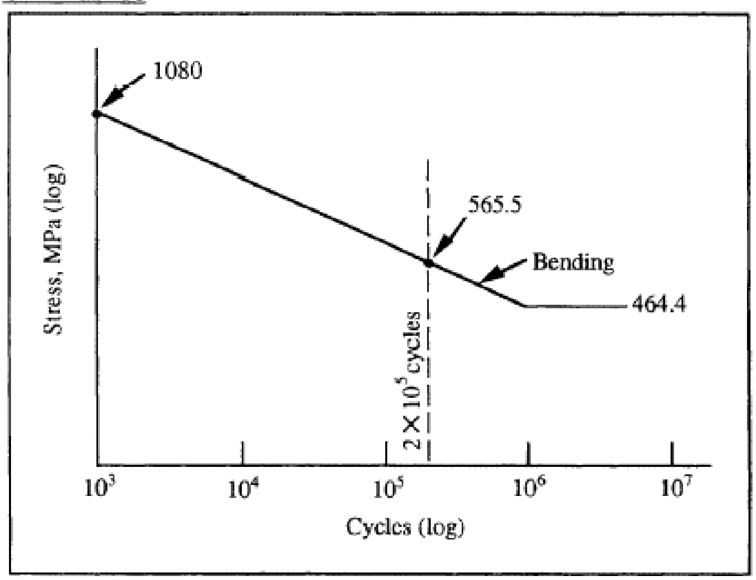
Find: Determine the fatigue strength for bending corresponding to (1) 10^6 or more cycles and (2) 2×10^5 cycles.

Schematic and Given Data:



Assumptions:
1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_S.
4. The gradient factor, $C_G = 0.9$.

Analysis:
1. Endurance limits: (10^6 cycle strength)
   $S_n = S_n' C_L C_G C_s$
   For bending,
   $S_n' = 0.5\ S_u = 0.5(1200) = 600$ MPa (Fig. 8.5)
   $C_L = 1$ (Table 8.1)
   $C_G = 0.9$ (Table 8.1)
   $C_s = 0.86$ (Fig. 8.13)
   $S_n = (600)(1)(0.9)(0.86) = 464.4$ MPa ■
2. 10^3 cycle strength
   For bending,
   $0.9S_u = 0.9(1200) = 1080$ MPa (Table 8.1)
3. S-N curves



4. 2×10^5 cycle strength
   Bending: 565.5 MPa ■

Comments:

1. The surface factor, C_s is not used for correcting the 10^3-cycle strength because for ductile parts the 10^3 strength is relatively unaffected by surface finish.
2. For critical designs, pertinent test data should be used rather than the preceding rough approximation.
3. Analytically the 200,000 cycle fatigue strength for bending may be determined by solving

   $$[\log(1080) - \log(565.5)]/(6 - 3) = [\log(S) - \log(565.5)]/(6 - \log(200{,}000)).$$

SOLUTION (8.25)

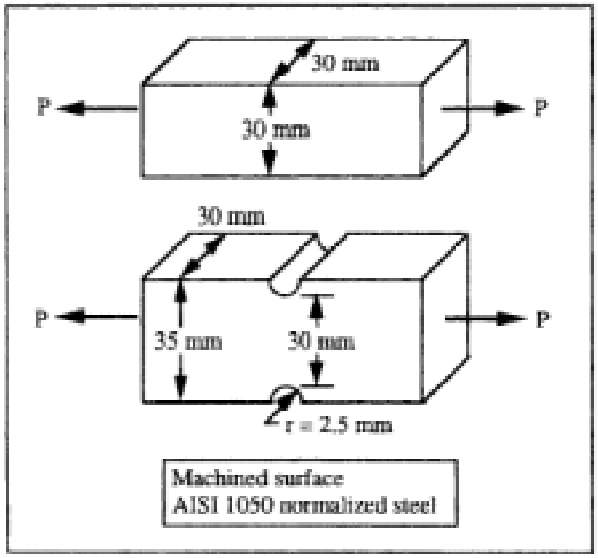
Known: An unnotched bar and a notched bar of known material have the same minimum cross section.

Find: For each bar, estimate

(a) the value of static tensile load P causing fracture

(b) the value of alternating axial load ± P that would be just on the verge of producing eventual fatigue fracture (after perhaps 1-5 million cycles).

Schematic and Given Data:



Assumption: The bar is manufactured as specified with regard to the critical fillet geometry and the bar surface finish.

Analysis:

1. For a static fracture of a ductile material, the notch has little effect. Hence, for both bars,

   $P = A \cdot S_u$

   where $S_u = 748.1$ MPa (Appendix C-4a)

   $P = (30 \text{ mm})^2 (748.1 \text{ MPa}) = 673 \times 10^3$ N

   $P = 670$ kN ■

2. $S_n = S_n' C_L C_G C_s$

   where $S_n' = 0.5S_u = 0.5(748.1)$ MPa

   $C_L = 1$ (Table 8.1)

   $C_G = 0.8$ (Table 8.1)

$C_s = 0.74$ (Fig. 8.13)

$S_n = 0.5(748.1)(1)(0.8)(0.74) = 221$ MPa

From Fig. 4.39, $K_t = 2.50$

Assuming Bhn = 217 (Appendix C-4a), using Fig. 8.24, $q \approx 0.86$

Thus, $K_f = 1 + (K_t - 1)q$ [Eq. (8.2)]

$K_f = 1 + (1.50)(0.86) = 2.29$

3. For the unnotched bar,

$P = A \cdot S_n = (30 \text{ mm})^2 (221 \text{ MPa})$

$= 199 \times 10^3 \text{ N} = 199 \text{ kN}$ ■

4. For the notched bar,

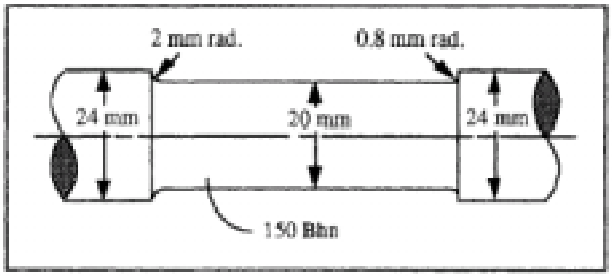
$P = A \cdot S_n / K_f = 199 \text{ kN}/2.29 = 87 \text{ kN}$ ■

SOLUTION (8.28)

Known: A machined shaft having a known hardness experiences completely reversed torsion.

Find: With a safety factor of 2, estimate the value of reversed torque that can be applied without causing eventual fatigue failure.

Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to the critical shaft geometry.

Analysis:

1. For steel,

$$S_u = 0.5 \text{ Bhn} = 0.5(150) = 75 \text{ ksi}$$

or, $S_u = 75 \text{ ksi}\left(\dfrac{6.890 \text{ MPa}}{\text{ksi}}\right) = 517 \text{ MPa}$

2. $S_n = S_n' C_L C_G C_s$

$S_n' = 0.5S_u = 0.5(517)$ (Fig. 8.5)
$C_L = 0.58$ (Table 8.1)
$C_G = 0.9$ (Table 8.1)
$C_s = 0.78$ (Fig. 8.13)

$S_n = 0.5(517)(0.58)(0.9)(0.78) = 105.3 \text{ MPa}$ ■

3. At the critical point (0.8 mm radius), $r/d = 0.04$ and $D/d = 1.2$
From Fig. 4.35(c), $K_t = 1.65$
From Fig. 8.23, $q = 0.74$
Hence, $K_f = 1 + (K_t - 1)q$ [Eq.(8.2)]
$= 1 + (0.65)(0.74) = 1.48$

4. Therefore, the nominal value of reversed torsional stress can be $\tau = 105.3/1.48 = 71.1$ MPa.

But, $\tau = \dfrac{16T}{\pi d^3}$ or $T = \dfrac{\tau \pi d^3}{16}$

$$T = \frac{(71.1 \text{ MPa})\pi(20 \text{ mm})^3}{16} = 111{,}700 \text{ N·mm}$$

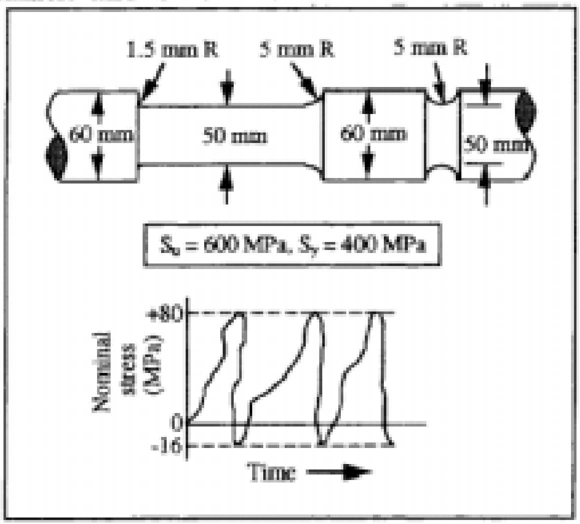
with SF = 2, $T = \dfrac{111.7 \text{ N·m}}{2} = 55.8 \text{ N·m}$ ■

SOLUTION (8.38)

Known: A shaft is subjected to a fluctuating nominal stress. The shaft is made of steel having known S_u and S_y.

Find: Estimate the safety factor with respect to eventual fatigue failure if:
(a) the stresses are bending,
(b) the stresses are torsional.

Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to surface finish and critical fillet radii.

Analysis:

1. For bending stresses,

$S_n = S_n' C_L C_G C_s$ [Eq. (8.1)]

$S_n' = 0.5S_u$ (Fig. 8.5)
$C_L = 1$ (Table 8.1)
$C_G = 0.9$ (Table 8.1)
$C_s = 0.77$ (Fig. 8.13)

$S_n = 0.5(600)(1)(0.9)(0.77) = 208$ MPa ■

2. Highest stress is at the 1.5 mm fillet where

$D/d = 1.2$ and $r/d = 0.03$

From Fig. 4.35, $K_t = 2.3$
From Fig. 8.24, $q = 0.78$
From Fig. (8.2), $K_f = 1 + (K_t - 1)q$

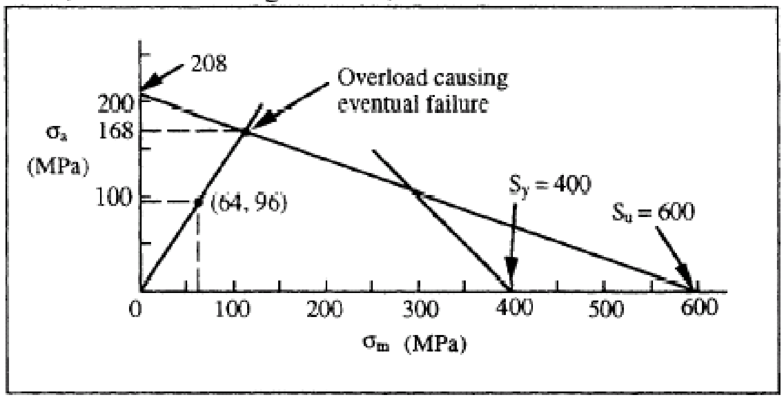
$K_f = 1 + (1.3)(0.78) = 2.01$

3. At the fillet

$$\sigma_m = 2.01\left(\frac{80 - 16}{2}\right) = 64 \text{ MPa}$$

$$\sigma_a = 2.01\left(\frac{80 + 16}{2}\right) = 96 \text{ MPa}$$

4. Thus, for the bending stresses,



SF = 168/96 = 1.8 ■

5. For torsional stresses,

$S_n = S_n' C_L C_G C_s$

$S_n' = 0.5S_u$

$C_L = 0.58$

$C_G = 0.9$

$C_s = 0.77$

$S_n = 0.5(600)(0.58)(0.9)(0.77) = 121$ MPa ■

6. From Fig. 4.35, $K_t = 1.78$

From Fig. 8.24, $q = 0.81$

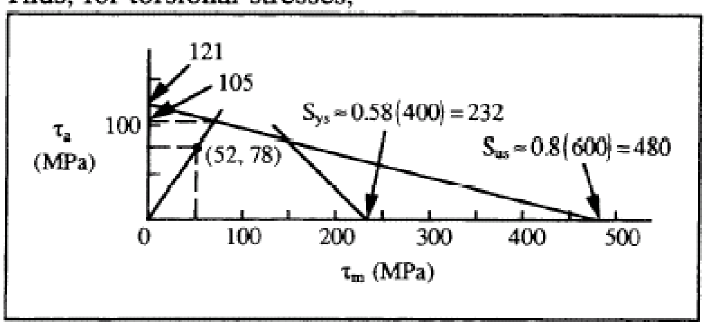
$K_f = 1 + (1.78 - 1)(0.81) = 1.63$

7. At critical fillet,

$$\tau_m = 1.63\left(\frac{80 - 16}{2}\right) = 52 \text{ MPa}$$

$$\tau_a = 1.63\left(\frac{80 + 16}{2}\right) = 78 \text{ MPa}$$

8. Thus, for torsional stresses,



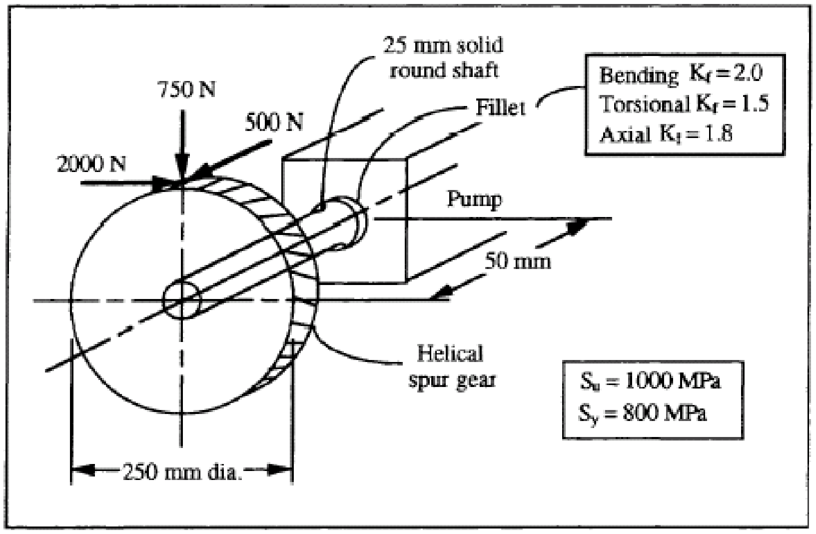
SF = 105/78 = 1.3 ■

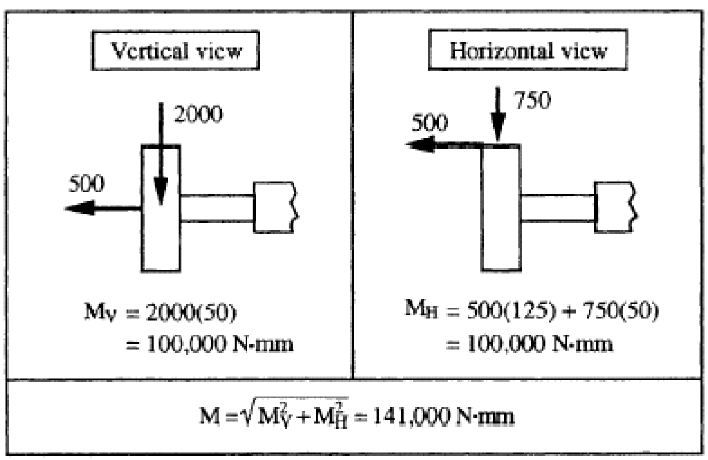
SOLUTION (8.44)

Known: A pump is gear-driven at uniform load and speed. The shaft is supported by bearings mounted in the pump housing. The shaft is made of steel having known values of S_u and S_y. The tangential, axial, and radial components of force applied to the gear are known. The surface of the shaft fillet has been shot-peened, which is estimated to be equivalent to a laboratory mirror-polished surface. Fatigue stress concentration factors for the fillet have been determined.

Find: Estimate the safety factor with respect to eventual fatigue failure at the fillet.

Schematic and Given Data:





Assumption: The shaft is manufactured as specified with regard to shaft geometry and surface finish.

Analysis:

1. $M_V = (2000)(50) = 100{,}000$ N·mm, $M_H = 500(125) + 750(50) = 100{,}000$ N·mm; $M = \sqrt{M_V^2 + M_H^2} = 141{,}000$ N·mm
2. We use the Fig. 8.16 relationship for "general biaxial loads":
   Alternating stress:

$$\sigma_a = \frac{32M}{\pi d^3}K_f = \frac{32(141{,}000)}{\pi(25)^3}(2) = 183.8 \text{ MPa}$$

$\sigma_{ea} = 183.8$ MPa

Mean stresses:

$$\tau = \frac{16M}{\pi d^3}K_f = \frac{16(2000)(125)}{\pi(25)^3}(1.5) = 122.2 \text{ MPa}$$

$$\sigma = \frac{P}{A}K_f = \frac{500(4)}{\pi(25)^2}(1.8) = 1.83 \text{ MPa}$$

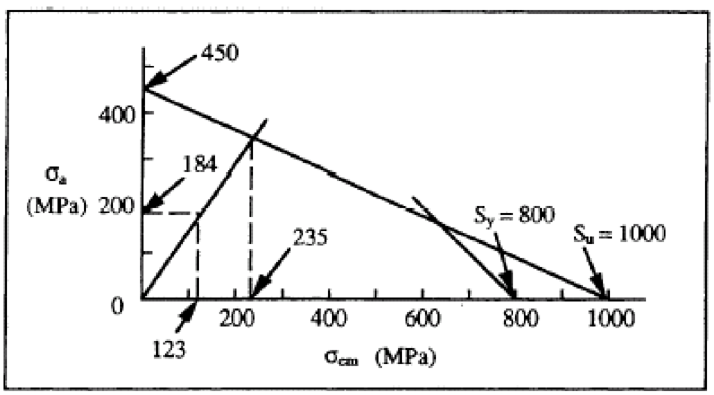
$$\sigma_{em} = \frac{\sigma}{2} + \sqrt{\tau^2 + \left(\frac{\sigma}{2}\right)^2} = .92 + \sqrt{122.2^2 + 0.92^2}$$

$\sigma_{em} = 123.1$ MPa

3. $S_n = S_n' C_L C_G C_s$ [Eq. (8.1)]
   - $S_n' = 0.5S_u$ (Fig. 8.5)
   - $C_L = 1$ (Table 8.1)
   - $C_G = 0.9$ (Table 8.1)
   - $C_s = 1$ (Fig. 8.13)

   $S_n = 0.5(1000)(1)(0.9)(1) = 450$ MPa

4.



5. SF = 235/123 = 1.9 ■

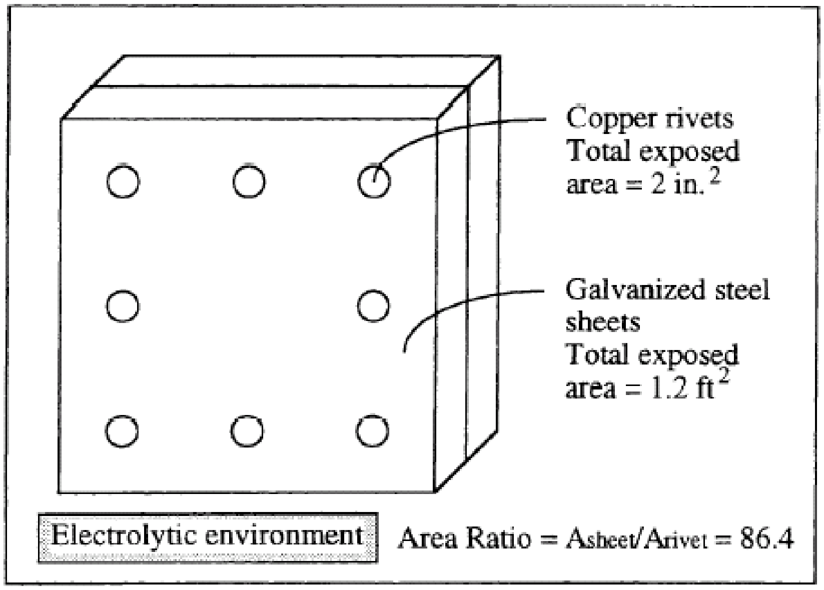
SOLUTION (9.3)

Known: Galvanized steel sheet metal of known area is riveted with copper rivets of known area. The entire assembly is immersed in salt water.

Find:

(a) Determine which metal will corrode.
(b) Determine the influence of using half as many rivets on the total rate of corrosion.
(c) Suggest ways to reduce corrosion.

Schematic and Given Data:



Assumptions:

1. Good electrical contact exists between the rivets and the galvanized metal sheets.
2. Salt is present in the environment.
3. The compatibility chart, Table 9.1, applies to this case.

Analysis:

(a) Since the galvanized metal #19 is more anodic than the copper #8, it will corrode (slowly, because of the area effect).
(b) Halving the number of copper rivets would about halve the corrosion rate of the galvanized steel sheets, because the current density at each copper rivet would remain about the same, thereby halving the current density of the galvanized metal.
(c) Reduce the corrosion by eliminating the electrolytic salt ions in the environment.

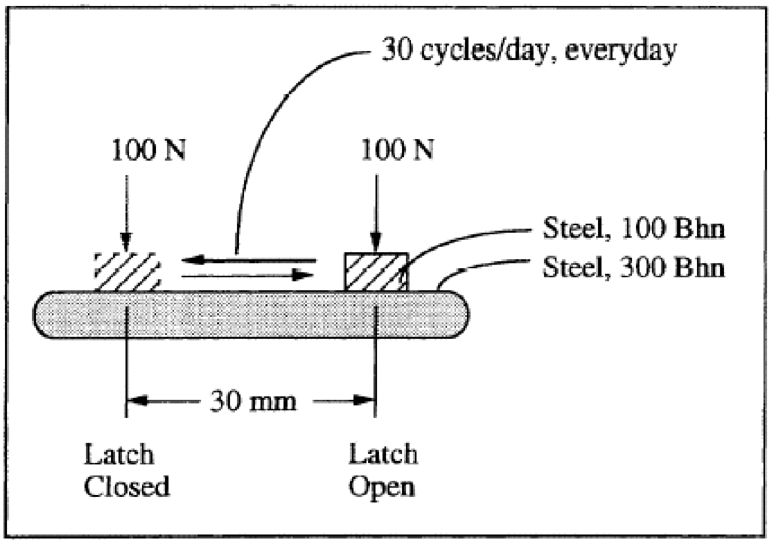
Comment: It is impossible to reduce the corrosion rate by using a sacrificial anode, since the galvanized steel is so anodic. Only magnesium is more active than galvanized metal but magnesium will react with air immediately.

SOLUTION (9.12)

Known: A latching mechanism involves steel mating surfaces of known hardness rubbing back and forth over a known distance. Lubrication is questionable. The latch is operated an average of 30 times per day. The compressive load between the surfaces is known.

Find: Estimate the volume of metal that will wear away from the softer steel member during one year of use.

Schematic and Given Data:



Analysis:

1. Apply Eq. (9.1a), $W = \left(\frac{K}{H}\right) FS$, where

   $K = 5 \times 10^{-3}$ (estimated from Fig. 9.12 for identical metals, poorly lubricated).
   $H = (100 \text{ Bhn})(9.81 \text{ MPa/Bhn}) = 981 \text{ MPa}$ for the softer steel.
   $F = 100 \text{ N}$
   $S = (2 \times 30 \text{ mm/operation})(30 \text{ operations/day}) \times (365 \text{ days/year}) = 657{,}000$ mm/year

2. $W = \dfrac{5 \times 10^{-3}}{981 \text{ MPa}} (100 \text{ N})(657{,}000 \text{ mm/year}) = 334.9 \text{ mm}^3\text{/year}$ ■

SOLUTION (10.1)

Known: A double-threaded Acme screw of known major diameter is used in a jack having a plain thrust collar of known mean diameter. Coefficients of running friction are estimated as 0.10 for the collar and 0.13 for the screw.

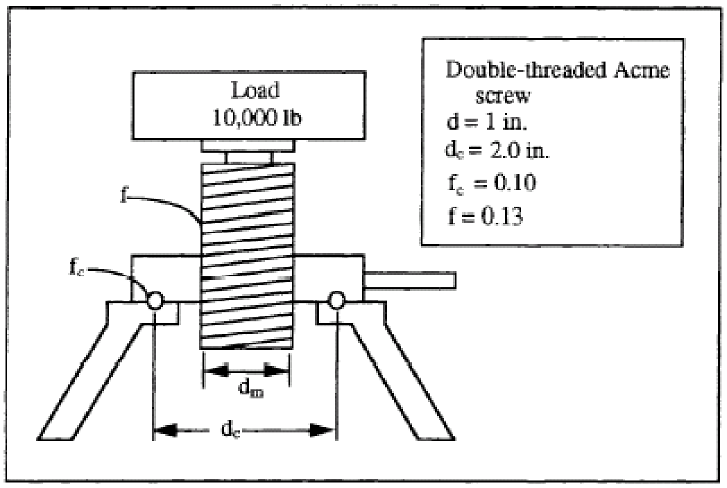
Find:

(a) Determine the pitch, lead, thread depth, mean pitch diameter, and helix angle of the screw.

(b) Estimate the starting torque for raising and for lowering a 10,000 lb load.

(c) If the screw is lifting a 10,000 lb load, determine the efficiency of the jack.

Schematic and Given Data:



Assumptions:

1. The starting friction is about 1/3 higher than running friction.
2. The screw is not exposed to vibration.

Analysis:

1. From Table 10.3, there are 5 threads per inch.

   $p = 1/5 = 0.2$ in. ■

   Because of the double-threaded screw,

   $L = 2p = 0.4$ in. ■

   From Fig. 10.4a,

   Threaded depth $= 0.5p = 0.10$ in. ■

   $d_m = d - 0.5p = 0.90$ in. ■

   From Eq. (10.1),

   $$\lambda = \tan^{-1}\left(\frac{L}{\pi d_m}\right) = \tan^{-1}\left(\frac{0.4}{0.90\pi}\right) = 8.05^o$$ ■

2. For starting, increase the given coefficients of friction by 1/3:

   $f_c = 0.133, f = 0.173$

   From Eq. (10.6),

   $\alpha_n = \tan^{-1}(\tan\alpha\cos\lambda) = \tan^{-1}(\tan 14.5^o \cos 8.05^o)$

   $= 14.36^o$

From Eq. (10.4),

$$T = \frac{Wd_m}{2}\left(\frac{f\pi d_m + L\cos\alpha_n}{\pi d_m\cos\alpha_n - fL}\right) + \frac{Wf_c d_c}{2}$$

$$= \frac{10{,}000(0.90)}{2}\left[\frac{0.173\pi(0.90) + 0.4\cos 14.36^\circ}{\pi(0.90)\cos 14.36^\circ - 0.173(0.4)}\right]$$

$$+ \frac{10{,}000(0.133)(2.0)}{2}$$

= 1477.5 + 1330 = 2807.5 lb•in. to raise the load ■

From Eq. (10.5),

$$T = \frac{Wd_m}{2}\left(\frac{f\pi d_m - L\cos\alpha_n}{\pi d_m\cos\alpha_n + fL}\right) + \frac{Wf_c d_c}{2}$$

$$= \frac{10{,}000(0.90)}{2}\left[\frac{0.173\pi(0.90) - 0.4\cos 14.36^\circ}{\pi(0.90)\cos 14.36^\circ + 0.173(0.4)}\right]$$

$$+ \frac{10{,}000(0.133)(2.0)}{2}$$

= 162.89 + 1330 = 1492.89 lb•in. to lower the load ■

3. From Eq. (10.4) with $f_c = 0.10$, $f = 0.13$

$$T = \frac{10{,}000(0.90)}{2}\left[\frac{0.13\pi(0.90) + (0.4)(\cos 14.36^\circ)}{\pi(0.90)\cos 14.36^\circ - 0.13(0.4)}\right] + \frac{10{,}000(0.10)(2.0)}{2}$$

= 1264.5 + 1000 = 2264.5 lb•in

4. From Eq. (10.4), the friction free torque for raising the load is

$$T = \frac{10{,}000(0.90)}{2}\left[\frac{(0.4)(\cos 14.36^\circ)}{\pi(0.90)\cos 14.36^\circ}\right] = 636.6 \text{ lb·in}$$

5. Efficiency = 636.6/2264.5 = 28% ■
6. Work input to the screw during one revolution = $2\pi T = 2\pi(2264.5)$ = 14228.8 lb•in
7. Work output during one revolution = WL = (10000)(2)(0.2) = 4000 lb•in
8. Efficiency = Work out/Work in = 4000/14228.8 = 28%

Comments:

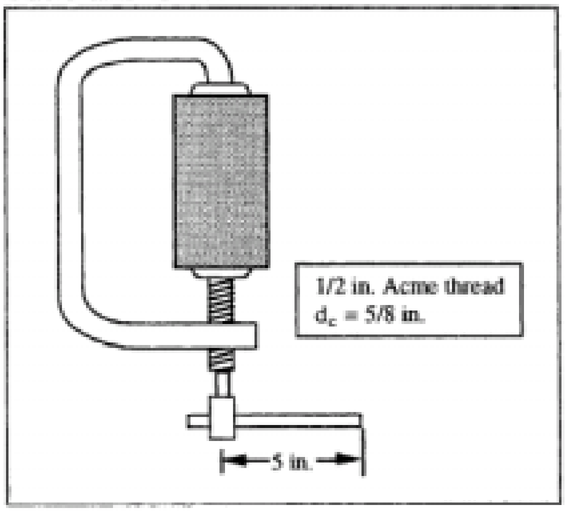
1. For a double threaded screw the work output during one revolution is WL where L = 2p.
2. If a small thrust bearing were used so that the collar friction could be neglected, the efficiency would increase to 636.6/1264.5 = 50%.

SOLUTION (10.9)

Known: An ordinary C-clamp uses a 1/2 in. Acme thread and a collar of 5/8 in. mean diameter.

Find: Estimate the force required at the end of a 5-in. handle to develop a 200 lb clamping force.

Schematic and Given Data:



Assumptions:

1. Coefficients of running friction are estimated as 0.15 for both the collar and the screw.
2. The screw has a single thread.

Analysis:

1. From section 10.3.1, and considering that service conditions may be conducive to relatively high friction, estimate $f = f_c \approx 0.15$ (for running friction).
2. From Table 10.3, p = 0.1 in., and with a single thread, L = 0.1 in.
3. From Fig. 10.4(a),

$$d_m = d - \frac{p}{2} = 0.5 - 0.05 = 0.45 \text{ in.}$$

$$\alpha = 14.5^\circ$$

From Eq. (10.1),

$$\lambda = \tan^{-1}\frac{L}{\pi d_m} = \tan^{-1}\frac{0.1}{\pi(0.45)} = 4.05^\circ$$

From Eq. (10.6),

$$\alpha_n = \tan^{-1}(\tan\alpha\cos\lambda) = \tan^{-1}(\tan 14.5^\circ \cos 4.05^\circ)$$
$$= 14.47^\circ$$

(Note: with $\lambda \approx 4^\circ$, it is obvious that $\alpha_n \approx \alpha$ and well within the accuracy of assumed friction coefficients)

4. From Eq. (10.4),

$$T = \frac{Wd_m}{2}\left(\frac{f\pi d_m + L\cos\alpha_n}{\pi d_m\cos\alpha_n - fL}\right) + \frac{Wf_c d_c}{2}$$

$$= \frac{(200)(0.45)}{2}\left(\frac{(0.15)\pi(0.45) + 0.1(\cos 14.47^\circ)}{\pi(0.45)(\cos 14.47^\circ) - (0.15)(0.1)}\right) + \frac{(200)(0.15)(0.625)}{2}$$

$$= 10.27 + 9.37 = 19.64 \text{ lb in.} \quad \text{Use } T \approx 20 \text{ lb in.}$$

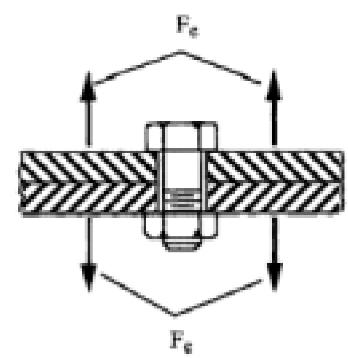
At the end of a 5-in. handle, the clamping force required $\approx \frac{20}{5} = 4$ lb ■

SOLUTION (10.19)

Known: In a given assembly, two parts are clamped together by a bolt. The ratio of the clamped member stiffness and the bolt stiffness is given. the initial bolt tension and the range of the fluctuating external load are also given.

Find: Draw a graph (plotting force vs. time) showing three or four external load fluctuations, and corresponding curves showing the fluctuations in total bolt load and total joint clamping force.

Schematic and Given Data:



F_e = 0 to 6,000 lb
$k_c = 6k_b$
F_i = 1,100 lb

Assumption: The bolt size and material are such that the bolt load remains within the elastic range.

Analysis:

1. The total bolt load when an external load is applied is, from Eq. (10.13),

$$F_b = F_i + \left(\frac{k_b}{k_b + k_c}\right) F_e = 1100 + \frac{1}{1+6}(6000)$$

$$= 1957 \text{ lb}$$

$$F_c = F_i - \left(\frac{k_c}{k_c + k_b}\right) F_e = 1100 - \frac{6}{7}(6000)$$

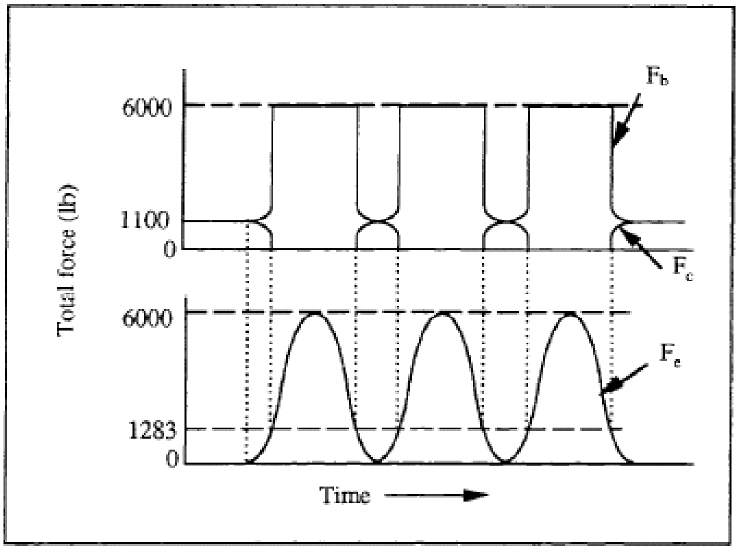
$$= -4043 \text{ lb, since } -4043 < 0,\ F_c = 0 \text{ lb and } F_b = 6000 \text{ lb}$$

2. When $F_c = 0$, separation takes place: $1100 - \frac{6}{7}F_e = F_c = 0$ and thus

   $F_e = \frac{7}{6}(1100) = 1283$ lb

3. With no external load: $F_b = F_c = F_i$

4.



SOLUTION (10.34)

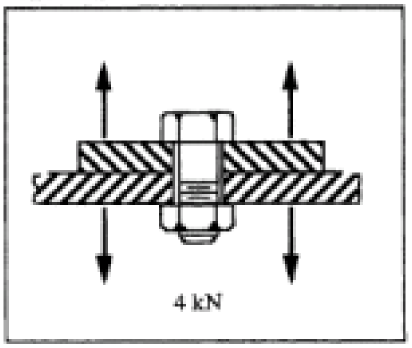
Known: The bolts that attach a bracket to an industrial machine must each carry a static tensile load of 4 kN. The safety factor is 5. The nuts are made of a steel with 2/3 the yield strength and proof strength of the bolt steel.

Find:

(a) Determine the size of class 5.8 coarse-thread metric bolts required.

(b) Determine the least number of threads that must be engaged for the thread shear strength to be equal to the bolt tensile strength.

Schematic and Given Data:



Assumptions:

1. For steel, $S_{ys} \approx 0.58S_y$.
2. The loads are equally distributed among the threads.
3. $(S_y)_{nut} = 2/3\ (S_y)_{bolt}$

Analysis:

(a) From Table 10.5, $S_p = 380$ MPa

$$A_t = \frac{(\text{Force})(\text{SF})}{S_p} = \frac{4{,}000(5)\ \text{N}}{380\ \text{MPa}} = 52.6\ \text{mm}^2$$

From Table 10.2, select M10 × 1.5 ■

(b) Bolt tensile strength $\approx A_t\,(S_y)_{bolt} = 58\ \text{mm}^2 \bullet (S_y)_{bolt}$

Nut shear strength $= \pi d(0.75t)S_{ys}$

$= \pi(10\ \text{mm})(0.75t)(0.58)(2/3)(S_y)_{bolt}$

where S_{ys} pertains to the nut material and $(S_y)_{bolt}$ to the bolt material.

Equating the strengths gives $t = 6.37$ mm

For pitch = 1.50 mm, this corresponds to $4.25 \approx 4.3$ thread ■

SOLUTION (10.39)

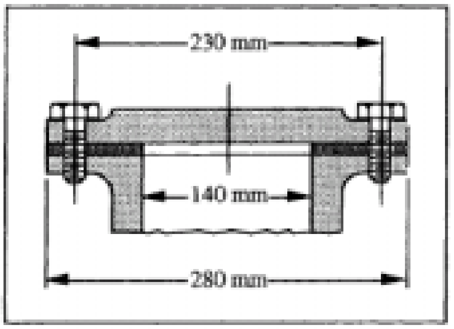
Known: The internal pressure of a pressure vessel with a gasketed end plate is sufficiently uniform that the bolt loading can be considered static. A gasket clamping pressure of at least 13 MPa is needed.

Find:

(a) For 12-, 16-, and 20-mm bolts with coarse threads and made of SAE class 8.8 or 9.8 steel, determine the number of bolts needed.

(b) If the ratio of bolt circle circumference to number of bolts should not exceed 10, nor be less than 5, state which of the bolt sizes considered gives a satisfactory bolt spacing.

Schematic and Given Data:



Assumption: When calculating gasket area, the bolt hole area is negligible.

Analysis:

(a) Clamping force required $= \frac{\pi}{4}\left(D_o^2 - D_i^2\right)$ (pressure)

$$= \frac{\pi}{4}\left(280^2 - 140^2\right) 13 = 600{,}358 \text{ N}$$

Bolt dia.	A_t (Tab. 10.2)	S_p (Tab. 10.5)	Clamping force/ bolt @ 90 % proof load $A_t(0.9\,S_p)$	600 kN force/bolt	Number of bolts required
12 mm	84.3 mm^2	650 MPa	49.3 kN	12.17	13
16	157	650	91.8	6.54	7
20	245	600	132.3	4.54	5

(b) For 12 mm bolts:

spacing = 230π/13 = 55.58 mm = 4.63d

For 16 mm bolts:

spacing = 230π/7 = 103.2 mm = 6.45d

For 20 mm bolts:

spacing = 230π/5 = 144.5 mm = 7.23d

The 16 and 20 mm bolts satisfy the given guidelines;
The 12 mm bolts are a little too close together. ■

SOLUTION (10.42)

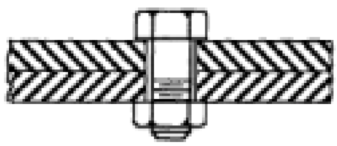
Known: A grade 5, 1-in. diameter UNF bolt has rolled threads, and is used to clamp two rigid members together such that $k_c = 4k_b$. There is a force tending to separate the members that fluctuates rapidly between 0 and 20,000 lb. There is a possibility of slight bolt bending. Two values of initial tension are to be investigated: (1) the normal value that a mechanic might tend to apply [Eq. (e) in Section 10.7], and (2) the theoretical limiting value of $A_t S_y$.

Find:

(a) Make a force vs. time plot for each of the two values of initial tension.

(b) Estimate the safety factor corresponding to each of the values of initial tension, where failure is considered to be either eventual fatigue fracture or joint opening (clamping force dropping to zero).

Schematic and Given Data:



Grade 5
1-in. UNF bolt
rolled threads
$k_c = 4k_b$
$F_e = 0$ to 20,000 lb

Assumption: The material has an idealized stress-strain curve, with the change from elastic to plastic occurring at the yield strength.

Analysis:

(a) *Force vs. time plot for two values of initial tension:*

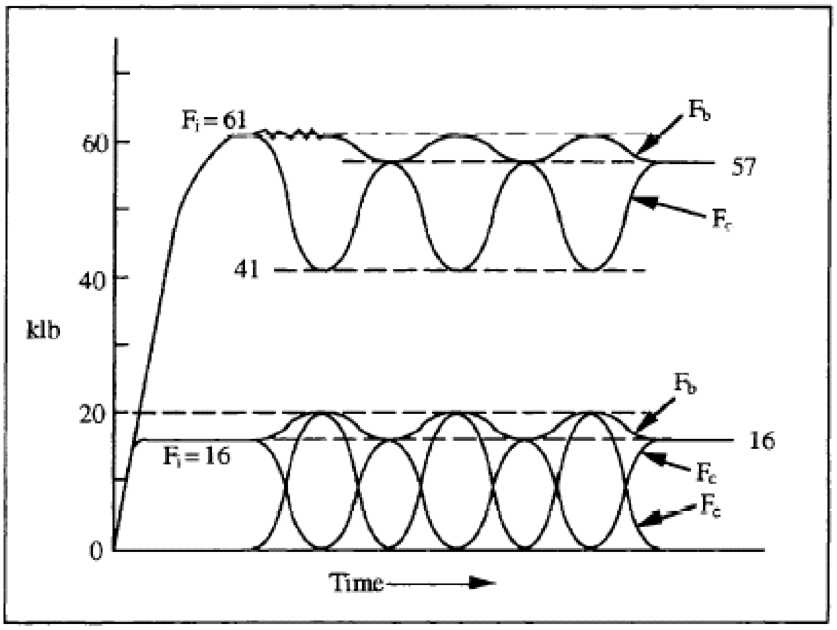
1. From Table 10.1, $A_t = 0.663$ in.^{2}
   From Table 10.4, $S_y = 92$ ksi
   From Table 10.6, $K_f = 3$
2. Eq. (e) tightening: $F_i = 16{,}000d = 16{,}000$ lb
   "Full tightening": $F_i = A_t S_y = (0.663)(92{,}000) = 61{,}000$ lb
3. Elastic response: $\dfrac{k_b}{k_b + k_c} = 1/5$

   (4/5 of load to clamped member)

   When bolt is plastic: $\dfrac{k_b}{k_b + k_c} = 0$

   (all of load to clamped member)

4.



(b) *Estimate the safety factor for joint opening:*

5. For F_e = 20,000 lb, F_c = 0 when F_i = 16,000 lb
   Thus, for F_i = 16,000 lb
   SF = 1
   and for F_i = 61,000 lb
   SF = 61,000/16,000 = 3.8

6. In estimating the safety factor for fatigue,

   $S_n = S_n' C_L C_G C_s$ [Eq. (8.1)]

   $S_n' = 0.5S_u$ (Fig. 8.5)
   S_u = 120 ksi (Table 10.4)
   $C_L = 1$ (Table 8.1)
   $C_G = 0.8$ (some bending)(Table 8.1)
   $C_s = 1$ (Table 10.6)

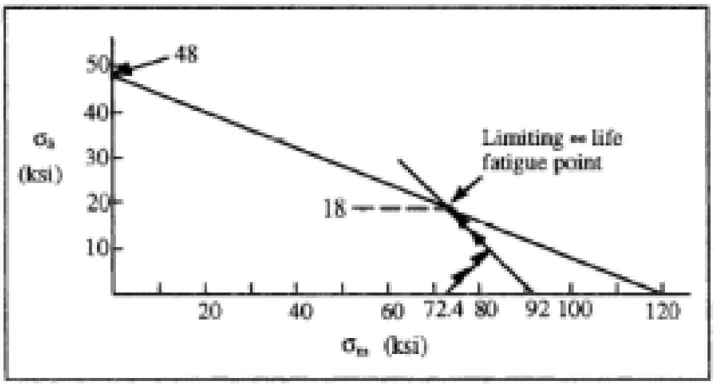
   $S_n = 0.5(120)(1)(0.8)(1) = 48$ ksi

7. For F_i = 16,000 lb,

   $$\sigma_i = \frac{16{,}000}{0.663}(3) = 72.4 \text{ ksi} \quad \text{(at thread root)}$$

   and an overload to failure proceeds as shown on the σ_m - σ_a plot to σ_a = 18 ksi
   For F_i = 61,000 lb, the thread root stresses proceed from $\sigma_m = S_y$, $\sigma_a = 0$ to the same limiting value of σ_a = 18 ksi.

8.



9. Since $F_a = 2000$ lb for the bolt, for either value of F_i,

$$\sigma_a = \frac{2000}{0.663}(3) = 9 \text{ ksi}$$

hence, SF = 18/9 = 2

10. For $F_i = 16{,}000$ lb, SF = 1, limited by separation. ■

For $F_i = 61{,}000$ lb, SF = 2, limited by fatigue. ■

SOLUTION (10.47)

Known: Solutions to problems (a) 10.42, (b) 10.43, (c) 10.44, (d) 10.45, and (e) 10.46 are given as the information in Section 10.12 and Table 10.7.

Find: Comment on the probable accuracy of the fatigue results. If previous designs had been made based on these earlier results, state whether or not it is important to specify that the bolt threads be rolled after heat treatment?

Analysis:

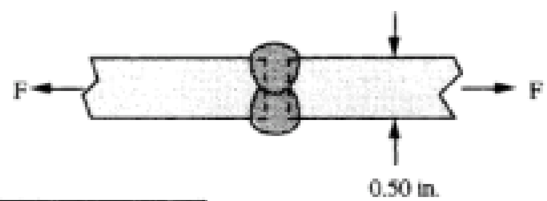
Base problem	S_u	Thread root σ_a	Nominal σ_a^*	Table 10.7 alt. strength roll before	Table 10.7 alt. strength roll after
(a) 10.33	120 ksi	18 ksi	6 ksi	10 ksi	21 ksi

SOLUTION (11.2)

Known: Two steel plates are butt welded using E60 series welding rods. The safety factor is 3.

Find: Determine the tensile load that can be applied to the plates per inch of welded plate width.

Schematic and Given Data:



S_y = 52.5 ksi (plate)
SF = 3
E60 series weld

Assumption: The weld efficiency is 100%.

Analysis:

1. From Section 11.4 the weld rod has a yield strength of $S_y = (60 - 12)$ ksi = 48 ksi. Since 48 ksi is less than S_y of the plates, the weld will yield first.
2. With the assumed 100% efficiency:

$$F = \frac{S_y A}{SF} = \frac{(48{,}000 \text{ psi})(0.50 \text{ in.})(L)}{3}$$

3. The maximum tensile load per inch of weld is F/L = 8000 lb/in. ■

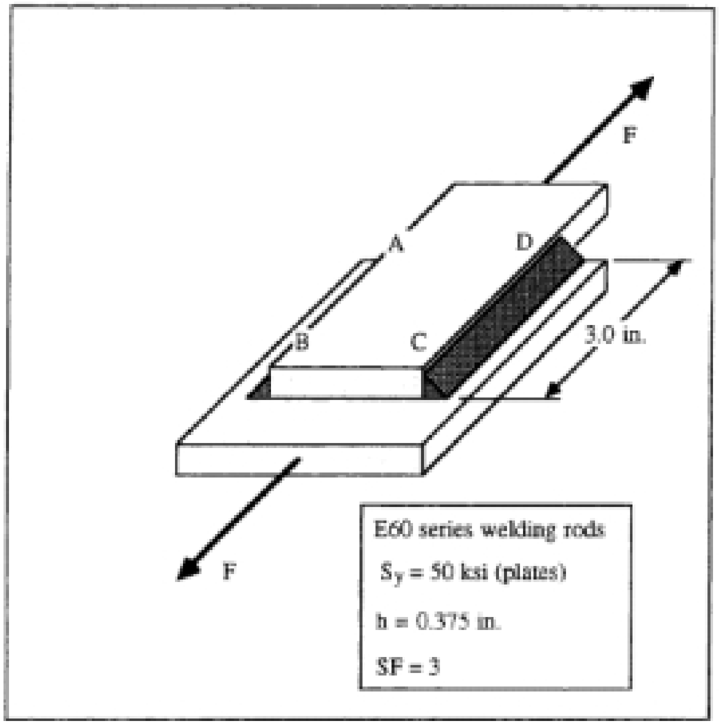
Comment: The load that the plate can carry per inch of length is given by $F = S_y A$ = (52,500)(0.50) = 26,250 lb which significantly exceeds that of the weld.

SOLUTION (11.7)

Known: Two steel plates are joined using 3/8 in. parallel-loaded fillet welds. The yield strength, S_y, and the length of the welds are known. The safety factor is 3.

Find: Determine the maximum tensile load that can be applied.

Schematic and Given Data:



Assumptions:

1. The throat length is given by t = 0.707 h.
2. The weld efficiency is 100%.

Analysis:

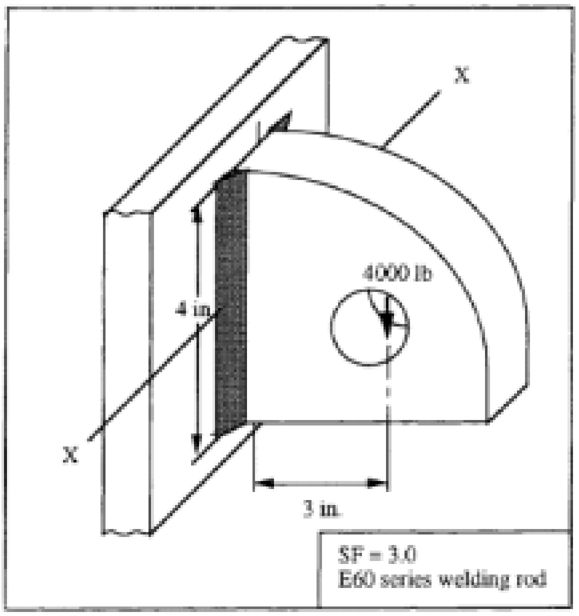
1. With t = 0.707 h, the throat area = (0.707)(3/8)(6) = 1.59 in.^{2}.
2. From Section 11.4, the yield strength of the weld material is $S_y = 60 - 12 = 48$ ksi.
3. Using the distortion energy theory, $S_{sy} = 0.58\ S_y = 0.58(48) = 27.8$ ksi.
4. Thus, $F = S_{sy}A/SF = (27{,}800)(1.59)/3 = 14{,}700$ lb. ■

SOLUTION (11.12)

Known: A bracket supports a 4000 lb load. A fillet weld extends for the full 4 in. length on both sides. Series E60 welding rod is used. The safety factor is 3.0.

Find: Calculate the minimum weld size required.

Schematic and Given Data:



Assumptions:

1. The throat length is given by t = 0.707 h.
2. The weld efficiency is 100%.

Analysis:

1. The stress due to direct shear is given by:
   $\tau = \frac{V}{A} = \frac{4000 \text{ lb}}{8t} = \frac{500}{t}$
2. The stress due to bending is given by:
   $\sigma = \frac{Mc}{I}$ where
   $M = (3 \text{ in.})(4000 \text{ lb}) = 12{,}000 \text{ lb.in}$

   $I = 2\left(\frac{L^3 t}{12}\right) = 2\left(\frac{4^3 t}{12}\right) = 10.67t \text{ in.}^4$; $c = 2$ in. Therefore, $\sigma = \frac{12{,}000(2)}{10.67\ t} = \frac{2249.3}{t}$
3. Vectorally adding σ and τ: Resultant stress $= \frac{1}{t}\sqrt{500^2 + 2249.3^2} = \frac{2304}{t}$

4. From Section 11.4 in the text, $S_y = 60 - 12 = 48$ ksi. Using the distortion-energy theory, $S_{ys} = 0.58\ S_y = 0.58(48) = 27.84$ ksi

5. $\dfrac{2304}{t} = \dfrac{S_{ys}}{SF} = \dfrac{27{,}840}{3.0}$; t = 0.25 in.

6. Since t = 0.707 h, $h = \dfrac{t}{0.707} = \dfrac{0.25}{0.707} = 0.35$ in. ■

SOLUTION (12.2)

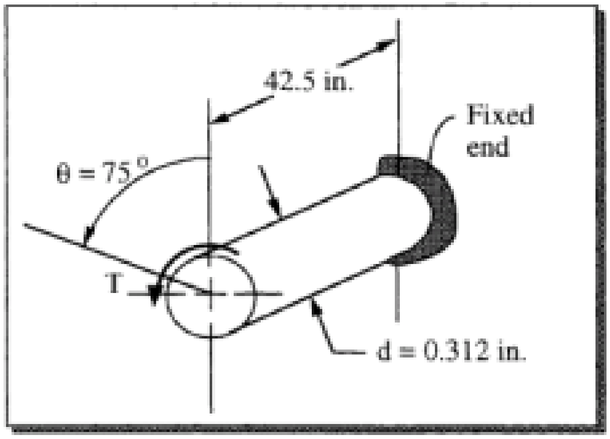
Known: A torsion bar has a known length and diameter.

Find:

(a) Estimate the change in shear stress when one end of the rod rotates through 75° relative to the other end.

(b) Estimate the change in torque.

Schematic and Given Data:



Assumptions:

1. The bar remains straight and the torque is applied about the longitudinal axis.
2. The material is homogeneous and perfectly elastic within the stress range involved.

Analysis:

1. From Table 5.1, for a torsional case (Case 2):

   $\theta = \dfrac{TL}{JG}$, therefore , $T = \dfrac{\theta JG}{L}$

2. $\theta = (75/180)\pi = 1.31$ rad

   $J = \pi d^4/32 = \pi(0.312)^4/32 = 0.00093$ in.^{4}

   From (Appendix C-1), $G = 11.5 \times 10^6$ psi

3. Therefore, $T = \dfrac{1.31 \times 0.00093 \times 11.5 \times 10^6}{42.5} = 329.66$ lb in. ■

4. From Eq. (4.4), for a solid round rod,

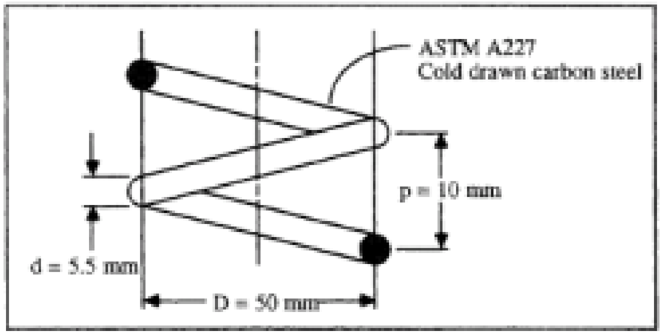
   $\tau = 16T/\pi d^3 = \dfrac{(16)(329.66)}{\pi(0.312)^3} = 55.3$ ksi ■

SOLUTION (12.10)

Known: A helical coil spring with given D and d is wound with a known pitch value. The material is ASTM A227 cold drawn carbon steel.

Find: If the spring is compressed solid, would you expect it to return to its original free-length when the force is removed?

Schematic and Given Data:



Assumptions:

1. There are no unfavorable residual stresses.
2. Both end plates are in contact with nearly a full turn of wire.
3. The end plate loads coincide with the spring axis.

Analysis:

1. Force to compress spring solid can be calculated by using Eq. (12.7).

$$F = \frac{d^4 G\delta}{8D^3 N}$$

where $\delta/N = p - d = 10 - 5.5 = 4.5$ mm

$G = 79 \times 10^9$ PA (Appendix C-1)

$$F = \frac{(5.5 \times 10^{-3})^4 (79 \times 10^9)(4.5 \times 10^{-3})}{8(50 \times 10^{-3})^3} = 325 \text{ N}$$ ■

2. The corresponding stress can be calculated by using Eq. (12.6).

$$\tau = \frac{8FD}{\pi d^3} K_s$$

for $C = D/d = 50/5.5 = 9.09$

$K_s = 1.05$ (Fig. 12.4)

$$\tau = \frac{8(325)(50 \times 10^{-3})}{\pi(5.5 \times 10^{-3})^3}(1.05) = 261 \text{ MPa}$$

3. From Eq. (12.9), $\tau_s \leq 0.45\ S_u$
   From Fig. 12.7, $S_u = 1300$ MPa
   Thus, $0.45\ S_u = 585$ MPa
4. Since 261 MPa < 585 MPa, no set should occur; therefore, spring should return to original length. ■

Comment: Even considering the curvature (stress concentration) factor of the inner surface by using $K_w = 1.17$, the inner surface stress is only 290 MPa which is still less than 585 MPa.

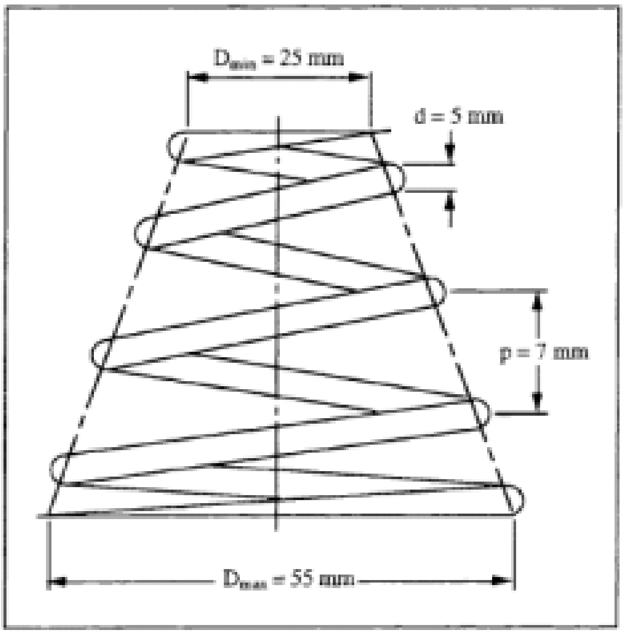
SOLUTION (12.14)

Known: A conical compression coil spring in Fig. 12.6 is made of steel wire. There are four active coils. A force is applied to compress the spring, and the stresses always remain in the elastic range.

Find:

(a) Which coil deflects to zero pitch first?
(b) Calculate the force required to cause the deflection in (a).
(c) Sketch a force vs. deflection for the spring.

Schematic and Given Data:



Assumptions:

1. There are no unfavorable residual stresses.
2. Both end plates are in contact with nearly a full turn of wire.
3. The end plate loads coincide with the spring axis.

Analysis:

1. From Eq. 12.8, $k = \frac{F}{\delta} = \frac{d^4 G}{8D^3 N}$

   Since $k \propto \frac{d^4}{D^3}$ and d is constant throughout, the largest active coil will have the smallest value of k. Thus, the largest (bottom) coil will deflect to zero pitch first.

2. From Eq. (12.7), $F = \dfrac{d^4 G \delta}{8D^3 N}$

where G = 79 GPa (Appendix C-1)

$$F = \frac{(5 \times 10^{-3})^4 (79 \times 10^9)(7 \times 10^{-3})}{8(55 \times 10^{-3})^3 (1)} = 259.67 \text{ N}$$

3. Flattening the smallest active coil requires

$$F = \frac{(5 \times 10^{-3})^4 (79 \times 10^9)(7 \times 10^{-3})}{8(25 \times 10^{-3})^3 (1)} = 2765.43 \text{ N}$$

Total deflection = p(N) = 7(5) = 35 mm

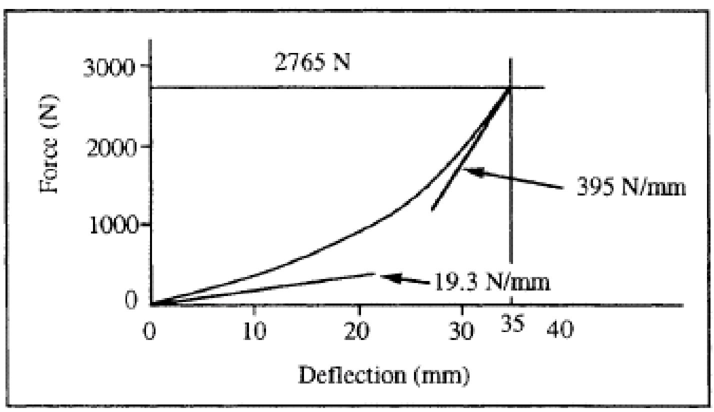
$$\text{Initial } k \approx \frac{d^4 G}{8D_{avg}{}^3 N} = \frac{(5 \times 10^{-3})^4 (79 \times 10^9)}{8(40 \times 10^{-3})^3 (5)} = 19{,}287 \text{ N/m}$$

or k = 19.3 N/mm

$$\text{Final } k \approx \frac{d^4 G}{8D_{min}{}^3 N} = \frac{(5 \times 10^{-3})^4 (79 \times 10^9)}{8(25 \times 10^{-3})^3 (1)} = 395{,}000 \text{ N/m}$$

or k = 395 N/mm

4.

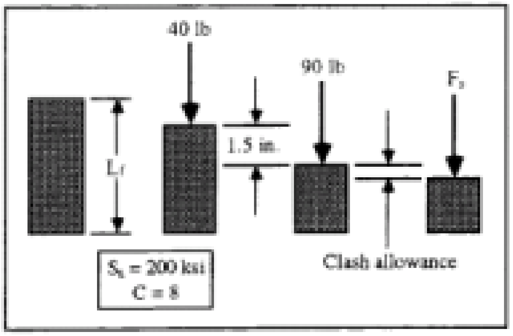


SOLUTION (12.21)

Known: A helical compression spring with squared and ground ends is to be made of steel, and presetting is to be used. The loading is static.

Find: Determine D, d, N, and L_f.

Schematic and Given Data:



Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.
3. The clash allowance is 10% of the maximum deflection.

Analysis:
1. From Eq. (12.9), $\tau_s = 0.65\ S_u = 0.65(200) = 130$ ksi.
2. With 10% clash allowance, $F_s = 1.10(90) = 99$ lb.
3. $k = \dfrac{\Delta F}{\Delta\delta} = \dfrac{50}{1.5} = 33.3$ lb/in.
4. From Eq. (12.6), $\tau_s = \dfrac{8F}{\pi d^2} CK_s$ where $K_s = 1.06$ (Fig. 12.4)

$$d = \left[\frac{8F_s}{\pi\tau_s}CK_s\right]^{1/2}$$

$$d = \left[\frac{8(99)}{\pi(130{,}000)}(8)(1.06)\right]^{1/2} = 0.128 \text{ in.} \qquad \blacksquare$$

5. $D = Cd = 8(0.128) = 1.02$ in. $\blacksquare$

6. From Eq. (12.8), $N = \dfrac{d^4G}{8D^3k}$ where $G = 11.5 \times 10^6$ psi (Appendix C-1)

$$N = \frac{(0.128)^4(11.5 \times 10^6)}{8(1.02)^3(33.3)} = 10.92 \qquad \blacksquare$$

7. From Fig. 12.8, for squared and ground ends,
$L_s = N_t d$ or $L_s = (N + 2)d$
$L_s = (10.92 + 2)(0.128) = 1.65$ in.

$$L_f = L_s + \delta_s = L_s + \frac{F_s}{k}$$

$$L_f = 1.65 + \frac{99}{33.3} = 4.62 \text{ in.} \qquad \blacksquare$$

Comment:

$$L_f/D = \frac{4.62}{1.02} = 4.53$$

$$\delta_s/L_f = \left(\frac{99}{33.3}\right)/4.62 = 0.644$$

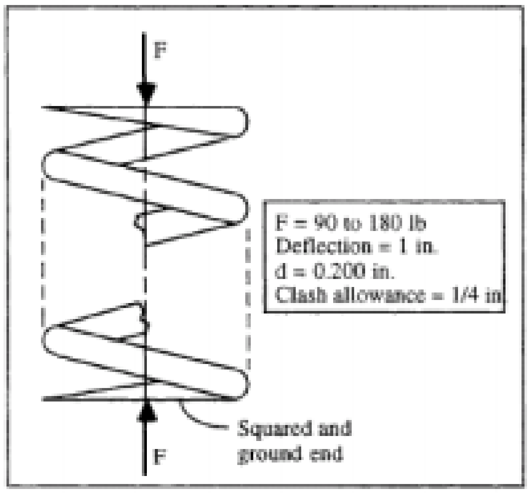
From Fig. 12.10, we can see that the end plates should be constrained parallel to avoid buckling.

SOLUTION (12.27)

Known: A coil spring with squared and ground ends is to operate with a load that fluctuates between 90 and 180 lb, during which the deflection is to vary by 1 in. Use a steel spring wire having d = 0.200 in. and fatigue strength properties as shown on Fig. 12.16 for shot-peened wire. Presetting, and a clash allowance of 1/4 in. are to be used. Residual stresses due to presetting are not to be taken into account.

Find: Determine appropriate values for N, D, and L_f.

Schematic and Given Data:



Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.

Analysis--Case A--with shotpeening:

1. From Fig. 12.16, for $\tau_{max}/\tau_{min} = 2$, τ_{max} = 800 MPa = 116 ksi

2. From Eq. (12.5), $\tau = \dfrac{8F}{\pi d^2} CK_w$ or $CK_w = \dfrac{\pi d^2 \tau_{max}}{8F_{max}}$

   $$CK_w = \frac{\pi(0.2)^2(116{,}000)}{8(180)} = 10.12$$

3. From Fig.12.4, C = 8.5
   D = Cd =8.5(0.2) = 1.70 in. ■

4. k = F/δ = 90/1 = 90 lb/in.

5. From Eq. (12.8), $k = \dfrac{dG}{8NC^3}$ or $N = \dfrac{dG}{8kC^3}$

   where G = 11.5 × 10^6 ksi (Appendix C-1)

   $$N = \frac{(0.2)(11.5 \times 10^6)}{8(90)(8.5)^3} = 5.2$$
   ■

6. $L_s = (N + 2)d = (5.2 + 2)(0.2) = 1.44$
Clash allowance = 0.25 in.
Working deflection = F/k = 180/90 = 2.00 in.
$L_f = 1.44 + 0.25 + 2.00 = 3.69$ in. ■

Analysis--Case B--without shotpeening:

1. From Fig. 12.16, for $\tau_{max}/\tau_{min} = 2$, $\tau_{max} = 640$ MPa = 93 ksi

2. From Eq. (12.5), $\tau = \frac{8F}{\pi d^2} CK_w$ or $CK_w = \frac{\pi d^2 \tau_{max}}{8F_{max}}$

$$CK_w = \frac{\pi(0.2)^2(93{,}000)}{8(180)} = 8.1$$

3. From Fig. 12.4, C = 6.5
$D = Cd = 6.5(0.2) = 1.30$ in. ■

4. $k = F/\delta = 90/1 = 90$ lb/in.

5. From Eq. (12.8), $k = \frac{dG}{8NC^3}$ or $N = \frac{dG}{8kC^3}$

where $G = 11.5 \times 10^6$ ksi (Appendix C-1)

$$N = \frac{(0.2)(11.5 \times 10^6)}{8(90)(6.5)^3} = 11.6$$ ■

6. $L_s = (N + 2)d = (11.6 + 2)(0.2) = 2.72$
Clash allowance = 0.25 in.
Working deflection = F/k = 180/90 = 2.00 in.
$L_f = 2.72 + 0.25 + 2.00 = 4.97$ in.

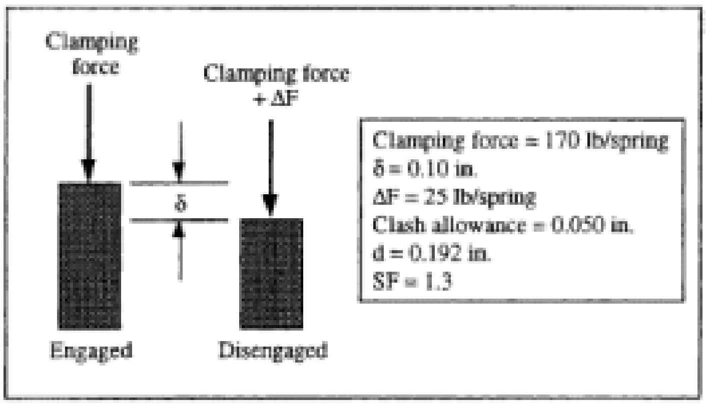
Comment: The answers given in the textbook are correct for non shot peened wire but are incorrect for shot peened wire.

SOLUTION (12.30)

Known: When the clutch is engaged, the springs must provide a clamping force of 1020 lb (170 lb per spring). When the clutch is disengaged the springs are all 0.10 in. shorter where a 25 lb increase of the force of each spring is considered satisfactory. The clash allowance is to be 0.050 in. The wire size is 0.192 in. The material is to be shot-peened wire having fatigue properties as indicated in Fig. 12.16. Use a safety factor for eventual fatigue failure of 1.3. Presetting is to be used but not taken into account in the calculations.

Find: Determine a suitable combination of D, N, L_s, and L_f.

Schematic and Given Data:



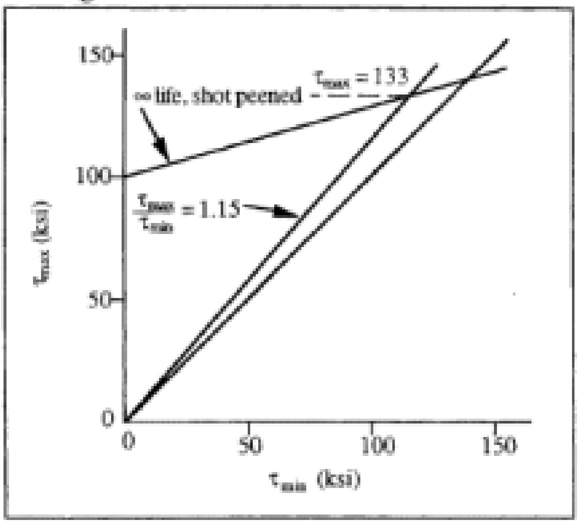
Assumptions:

1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.
3. The springs have squared and ground ends.

Analysis:

1. For $\frac{\tau_{max}}{\tau_{min}} = \frac{F_{max}}{F_{min}} = \frac{170 + 25}{170} = 1.15$

2. See Fig. 12.16



3. From above graph, $\tau_{max} = 133$ ksi

For SF = 1.3, use $\tau_{max} = \frac{133}{1.3} = 102$ ksi

4. From Eq. (12.5), $\tau_{max} = \frac{8F_{max}}{\pi d^2} CK_w$

$102{,}000 = \frac{8(195)}{\pi(0.192)^2} CK_w$

$CK_w = 7.57$

5. From Fig. 12.4, C = 6.0

$D = Cd = (6.0)(0.192) = 1.15$ in. ■

6. $k = \frac{F}{\delta} = \frac{25 \text{ lb}}{0.1 \text{ in.}} = 250$ lb/in.

7. From Eq. (12.8), $k = \frac{dG}{8C^3N}$ or $N = \frac{dG}{8C^3k}$

where $G = 11.5 \times 10^6$ ksi (Appendix C-1)

$N = \frac{(0.192)(11.5 \times 10^6)}{8(6.0)^3(250)} = 5.1$ ■

8. Assuming squared and ground ends,

$L_s = (N + 2)d = (7.1)(0.192) = 1.36$ in. ■

$L_f = L_s + \text{clash allowance} + \frac{F_{max}}{k} = 1.36 + 0.05 + \frac{195}{250} = 2.19$ in. ■

SOLUTION (12.41)

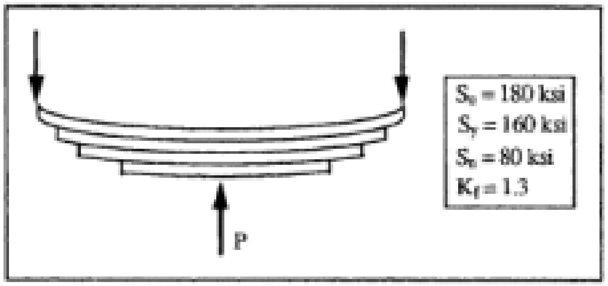
Known: A semi-elliptic leaf spring has four leaves, each made from 0.1 in. × 2 in. steel having known properties.

Find: Use the simplified "triangular plate" model.

(a) Determine the total spring length needed to give a spring rate of 75 lb/in.

(b) In service, the spring will carry a static load (applied to the center) of P, plus a superimposed dynamic load which varies from +P/2 to -P/2. Determine the highest value of P that will give infinite life, with a safety factor of 1.3.

Schematic and Given Data:



Assumptions:

1. The end points apply to a uniform load over the width of the spring ends.
2. Failure does not occur at the spring end.
3. The central force is aligned so as not to induce twisting in the spring.
4. The deflections do not significantly change the geometry.

Analysis:

1. With reference to Fig. 12.22b, $k = \dfrac{2F}{\delta} = \dfrac{Ebh^3}{3L^3}$

   where $E = 30 \times 10^6$ psi (Appendix C-1)

   $$75 = \frac{(30 \times 10^6)(2)(4)(0.1)^3}{3L^3}$$

   L = 10.2 in.

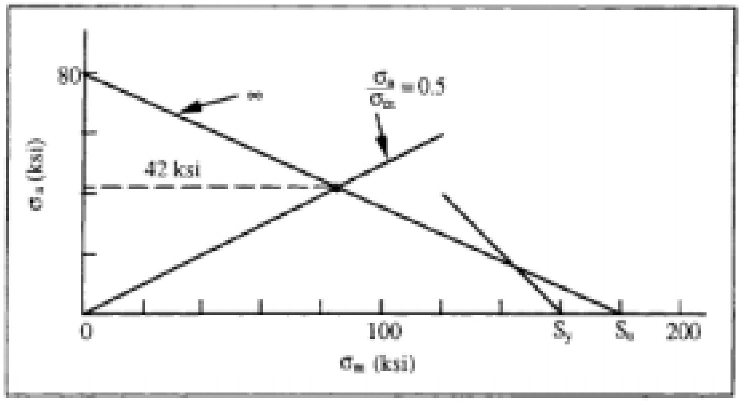
   Hence, spring length = 20.4 in. ■

2. From Fig. 12.22b, $\sigma = \frac{6FL}{bh^2} K_f$

$$\sigma_m = \frac{6(P/2)(10.2)}{8(0.1)^2}(1.3) = 497\ P$$

$$\sigma_a = \frac{6(P/4)(10.2)}{8(0.1)^2}(1.3) = 284.5\ P$$

3.



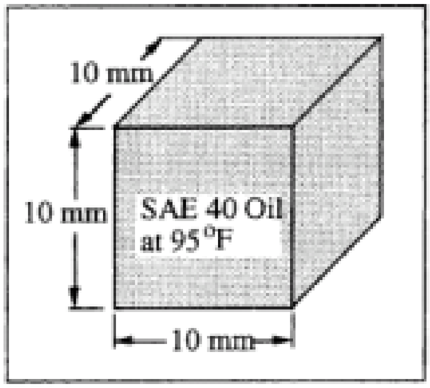
4. From graph, σ_a = 42 ksi = 248.5P; therefore, P = 169 psi. But with safety factor of 1.3, P = 169/1.3 or P = 130 lb. ■

SOLUTION (13.6)

Known: SAE 40 oil is at 95 °F.

Find: Determine the density in gram per cubic centimeter.

Schematic and Given Data:



Assumption: Equation 13.6a is valid.

Analysis:

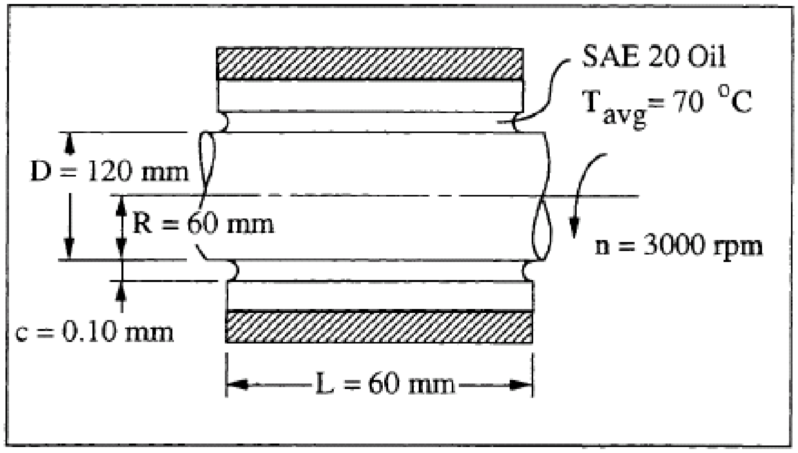
1. 95 °F = (95 - 32) (5/9) = 35 °C.
2. From Eq. 13.6a with T = 35 °C,
   $\rho = 0.89 - 0.00063(35 - 15.6) = 0.88\ \text{g/cm}^3$ ■

SOLUTION (13.16)

Known: A journal bearing has a known diameter, length, and diametral clearance. The journal has a given rotational speed and is lubricated with a SAE 20 oil at a known average temperature.

Find: Determine the power loss and the friction torque.

Schematic and Given Data:



Assumptions:
1. The effect of eccentricity between the journal bearing and the journal is negligible.
2. There is no lubricant flow in the axial direction.
3. The radial load is small.

Analysis:

With the above assumptions, Petroff's equation would be applicable.

1. From Fig. 13.6, μ = 12.5 mPa·s = .0125 Pa·s

$$= 0.0125\ \frac{\text{Ns}}{\text{m}^2}$$

2. From Eq. (b) on page 485, $T_f = \dfrac{4\pi^2 \mu n L R^3}{c}$

$$= \frac{4\pi^2 (0.0125 \frac{\text{Ns}}{\text{m}^2})(50 \text{ rev/s})(0.060\text{m})(0.060 \text{ m})^3}{0.00010 \text{ m}}$$

$T_f = 3.2$ N·m ■

3. From Eq. 1.2, $\dot{W} = \dfrac{nT}{9549}$

$$\dot{W} = \frac{3.2 \text{ N·m } (3000 \text{ rev/min})}{9549 \frac{\text{N·m rev}}{\text{min kW}}} = 1.01 \text{ kW}$$ ■

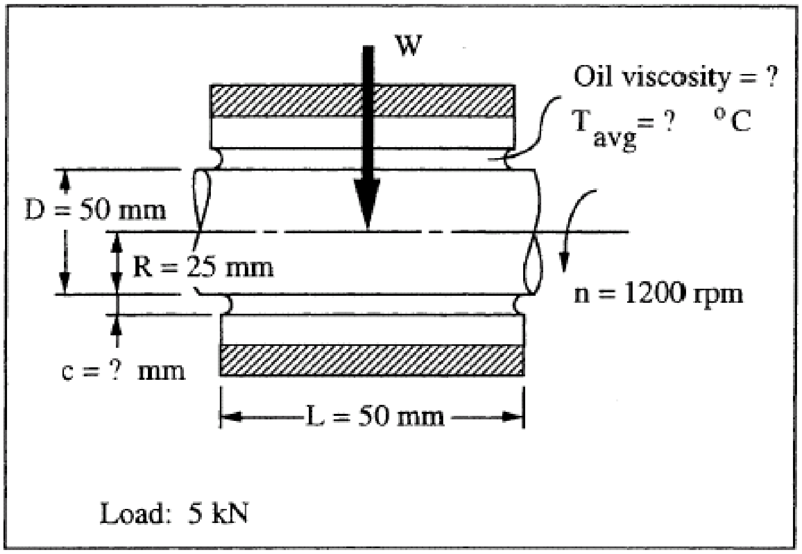
Comment: In an actual situation, we would need to verify that when dissipating 1.01 kW, the average oil temperature in the bearing would be consistent with the value of viscosity used in the calculations.

SOLUTION (13.24)

Known: A journal rotates at a given speed and supports a known radial load producing a minimum friction in a journal bearing with known diameter, length and minimum film thickness.

Find: Determine the radial clearance, oil viscosity, coefficient of friction and friction power loss.

Schematic and Given Data:



Assumptions:

1. The lubricant is supplied to the bearing at atmospheric pressure.
2. The influence on flow rate of any oil holes or grooves is negligible.
3. Viscosity is assumed to be constant, and to correspond to the average temperature of the oil flowing to and from the bearing.

Analysis:

1. From Fig. 13.13, the bearing is operating at $S = 0.08$, $h_o/c = 0.3$. Since $h_o = 0.025$ (given), $c = 0.025/0.3 = 0.083$ mm. ■

2. $$S = 0.08 = \left(\frac{R}{c}\right)^2 \frac{\mu n}{P} = \left(\frac{25}{0.083}\right)^2 \frac{\mu(20)}{5000/(0.050)^2}$$

   Hence, $\mu = 0.88 \dfrac{\text{Ns}}{\text{m}^2} = 0.088$ Pa·s = 88 mPa·s ■

3. From Fig. 13.14, $\frac{R}{c}f = 2.4$, hence $f = 2.4 \dfrac{0.083}{25} = 0.0080$ ■

4. Power loss = $fW(2\pi Rn)$ = (0.008)(5000 N) [2π(0.025 m)(20 rev/sec)] = 126 W (or 0.17 hp) ■

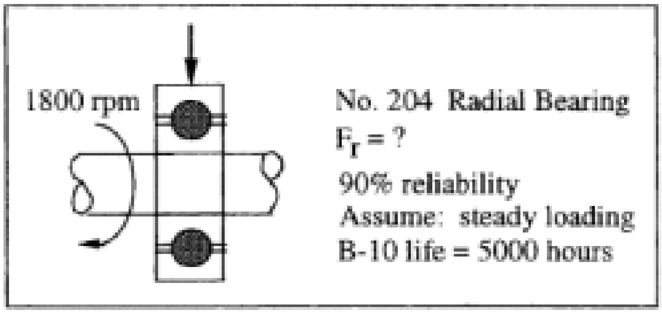
Comment: The foregoing analysis involving the Raimondi and Boyd charts applies only to steady-state operation with a load that is fixed in magnitude and direction.

SOLUTION (14.8)

Known: A No. 204 radial ball bearing has a 5000 hr B-10 life at 1800 rpm.

Find: Determine the bearing radial load capacity.

Schematic and Given Data:



Assumptions:

1. Table 14.2 accurately gives the bearing capacity.
2. Ball bearing life varies inversely with the 10/3 power of the load (i.e., Eq. (14.5a) is accurate).
3. The life given is for a 90% reliability.
4. The loading is steady.

Analysis:

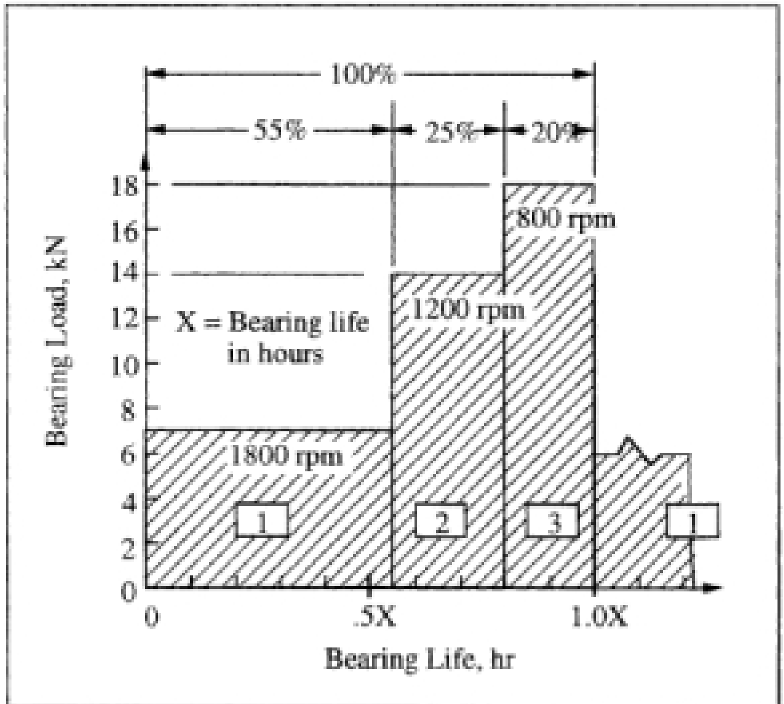
1. From Table 14.1, for a 204 bearing the bore is 20 mm.
2. From Table 14.2, for $L_R = 90 \times 10^6$ rev and a 200 series bearing, C = 3.35 kN.
3. From Fig. 14.13, for 90 percent reliability, $K_r = 1.0$.
4. From Table 14.3, $K_a = 1.0$ for a steady load.
5. From Eq. (14.5a), $L = K_r L_R (C/F_e K_a)^{3.33}$
6. Substituting and solving for F_e: $F_e = F_r = C(L_R/L)^{0.3}$
7. Substituting values:

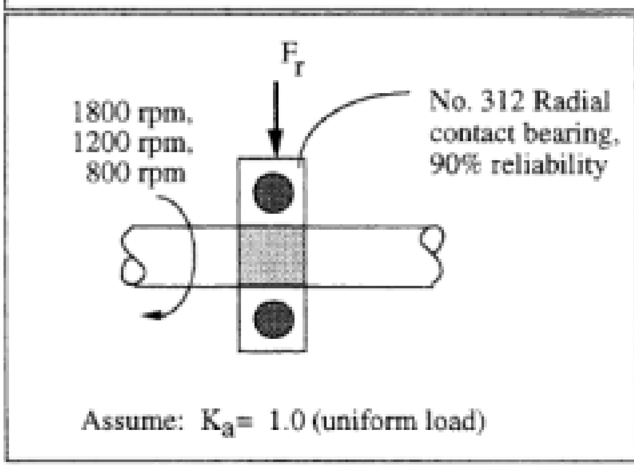
$$F_r = 3.35\text{ kN}\left[\frac{90 \times 10^6\text{ rev}}{(5000\text{ hr})(60\text{ min/hr})(1800\text{ rev/min})}\right]^{0.3} = 1957\text{ N} \quad \blacksquare$$

SOLUTION (14.15)
Known: A No. 312 radial contact ball bearing is loaded uniformly with three different loads and for three different periods.

Find: Estimate the bearing life for 90% reliability.

Schematic and Given Data: The load versus life (hr) diagram can be constructed from the given data:





Assumptions:

1. The change in load occurs without shock.
2. The bearing life varies inversely with the 10/3 power of the load.
3. Miner's rule is appropriate for this analysis.

Analysis:

1. From Table 14.2, for a No. 312 ball bearing, $C = 20$ kN for 90×10^6 revolution life with 90 percent reliability.
2. From Fig. 14.13, for 90% reliability, $K_r = 1.0$.
3. From Table 14.3, for no impact $K_a = 1.0$.
4. With $K_r = 1.0$, $L_R = 90 \times 10^6$, and $K_a = 1.0$, Eq. (14.5a) becomes
   $L = 90 \times 10^6(C/F_e)^{3.33}$
5. With the above equation, for $C = 20$ kN, we have for
   (a) $F_e = 7$ kN, $L = 2.968 \times 10^9$
   (b) $F_e = 14$ kN, $L = 2.952 \times 10^8$
   (c) $F_e = 18$ kN, $L = 1.278 \times 10^8$
6. Let X equal the total bearing life in hours.
7. The number of cycles at 7 kN, n_1 = (.55X hours)(1800 rev/min)(60 min/hr)
   = 59,400X cycles.
   Likewise, at 14 kN, n_2 = (.25X hours)(1200 rev/min)(60 min/hr)
   = 18,000X cycles.
   And at 18 kN, n_3 = (.20X hours)(800 rev/min)(60 min/hr)
   = 9600X cycles.
8. With $N_i = L_i$, $i = 1,2,3$, Eq. (8.3) becomes

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

$$\text{or} \quad \frac{59{,}400X}{2.968 \times 10^9} + \frac{18{,}000X}{2.952 \times 10^8} + \frac{9600X}{1.278 \times 10^8} = 1$$

Hence, X = 6406 hours ■

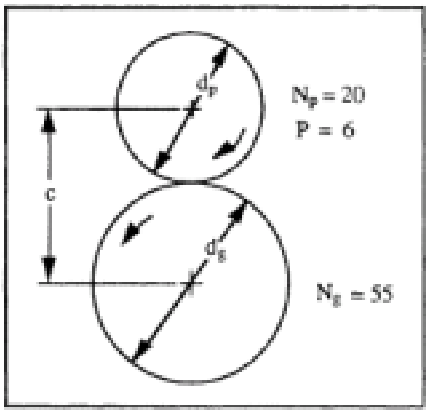
Comment: The cumulative damage of each of three loads is respectively, 13%, 39%, and 48%; i.e., (n_i/L_i), $i = 1,2,3$.

SOLUTION (15.4)

Known: A pinion with 20 teeth and 6 diametral pitch meshes with a gear having 55 teeth.

Find: Calculate the standard center distance.

Schematic and Given Data:



Assumptions:

1. The gears are spur gears.
2. The gears have teeth of standard involute profile.
3. The gears mesh along their pitch circles.

Analysis:

1. From Eq. (15.3): $P = N/d = N_p/d_p = N_g/d_g$
2. $P = 6$, $N_p = 20$. Hence, $d_p = 3.33$ in.
3. $N_g = 55$. Hence, $d_g = 9.17$ in.
4. The center distance, $c = \dfrac{d_g + d_p}{2} = 6.25$ in. ■

Comments:

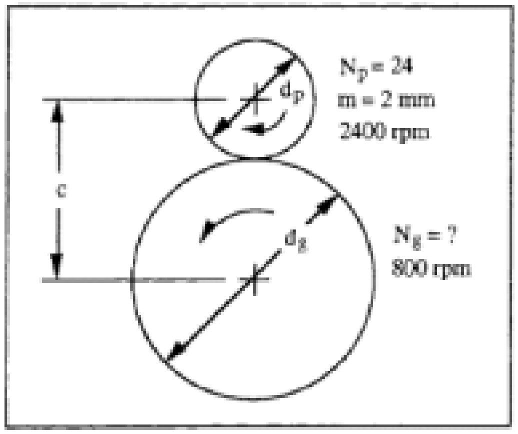
1. If the gears did not mesh at the theoretical pitch circles the measured (actual) center distance would not be equal to the sum of the theoretical pitch circle radii of the gears.
2. It should be evident that meshing gears must have the same diametral pitch.

SOLUTION (15.10)

Known: A pinion with known module and number of teeth rotates at 2400 rpm and drives a gear at 800 rpm.

Find: Determine the number of teeth on the gear, circular pitch and theoretical center distance.

Schematic and Given Data:



Assumption: The spur gears mesh along their pitch circles.

Analysis:

1. For the 3:1 velocity ratio, $N_g = 24(3) = 72$ ■
2. $p = \pi m = 2\pi$ mm ■
3. $d = Nm$; Hence $d_p = 48$ mm, $d_g = 144$ mm
4. $c = \dfrac{48 + 144}{2} = 96$ mm ■

Comments:

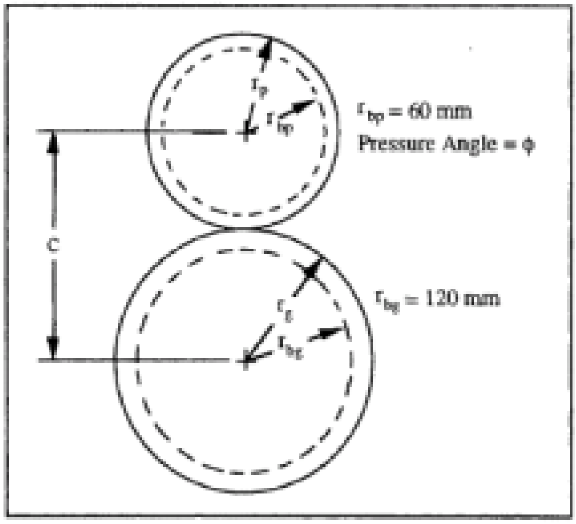
1. Similar to the diametral pitch, the module must be the same for a pair of meshing gears.
2. If the module were a higher value the pitch diameters of the gears and the theoretical center distance would have been higher (other parameter values remaining the same).

SOLUTION (15.13)

Known: A pair of involute gears of known base circle diameters with (a) center distance = 120 mm and (b) center distance = 100 mm is given.

Find: Determine the pressure angles of the gears for cases (a) and (b), and the ratio of pitch diameters.

Schematic and Given Data:



Assumption: The spur gears mesh along their pitch circles.

Analysis:

1. Let r_{bp} and r_{bg} represent pinion and gear base circle radii, respectively.

$$r_p = \frac{r_{bp}}{\cos\phi} \text{ and } r_g = \frac{r_{bg}}{\cos\phi}$$

In case (a), $c = r_p + r_g = 120 \text{ mm} = \frac{30}{\cos\phi} + \frac{60}{\cos\phi}$;

$\cos\phi = 0.75$, $\phi = 0.7227$ rad $= 41.4°$ ■

2. Similarly, in case (b), $\cos\phi = \frac{90}{100} = 0.9$, $\phi = 0.4510$ rad $= 25.8°$ ■

3. $d_g/d_p = d_{bg}/d_{bp} = 120/60 = 2$ (for any center dist.) ■

Comments:

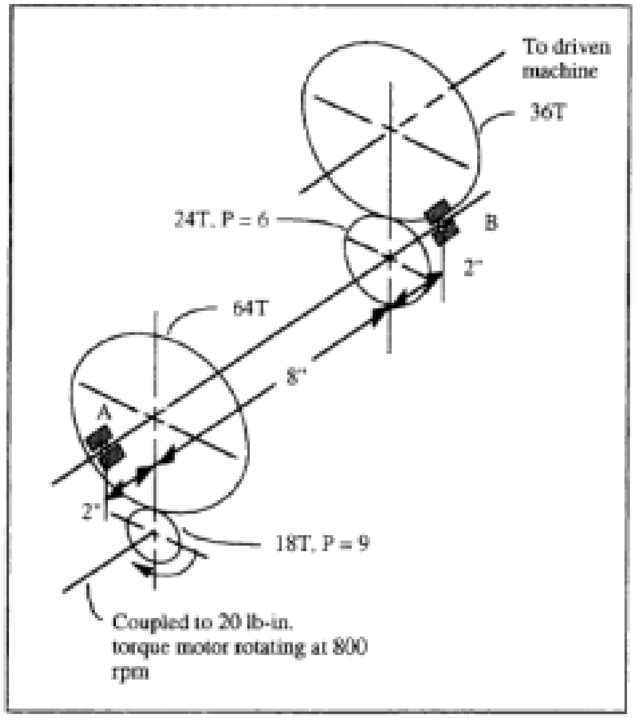
1. With fixed base radii, reduction in center distance resulted in a reduction in pressure angle.
2. Changes in the pressure angle for the gear pair did not affect the ratio of pitch diameters since the pressure angle must be the same for meshing gears and the ratio of base diameters is fixed in this case.

SOLUTION (15.23)

Known: A two stage spur gear speed reducer of specified geometry is given.

Find: Determine the radial loads applied to the countershaft bearings.

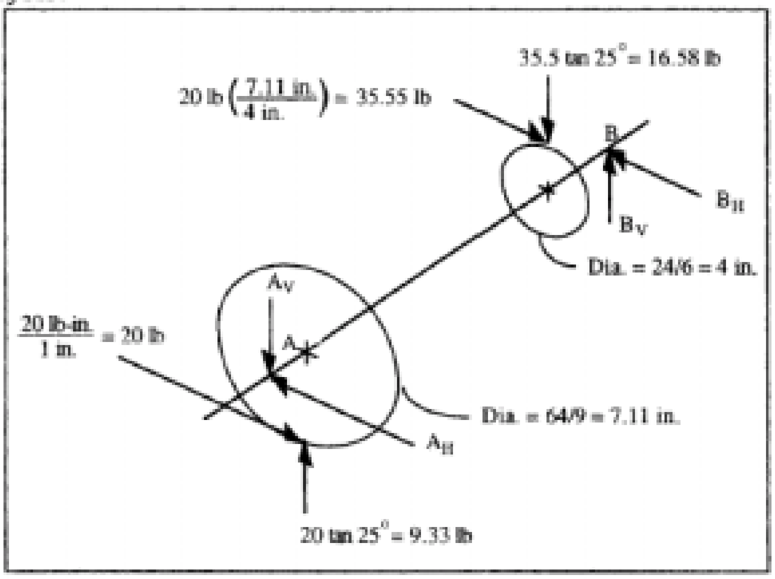
Schematic and Given Data:



Assumptions:

1. The gears mesh along their pitch circles.
2. Friction losses in gears and bearings can be neglected (given).
3. The shafts are all parallel.
4. All the gear radial and tangential load is transferred at the pitch point.
5. Bending deflection of the countershaft is negligible.
6. Gear face width and bearing widths are negligible relative to countershaft length.
7. The gears are rigidly connected to their shafts.

Analysis:



1. Pitch diameter of input pinion $= \frac{N}{P} = \frac{18}{9} = 2$ in.
2. For the horizontal plane:
   $\sum M_A = 0$:
   $20 \text{ lb } (2 \text{ in.}) + 35.55 \text{ lb } (10 \text{ in.}) - B_H(12 \text{ in.}) = 0$
   hence, $B_H = 32.96$ lb
   $\sum F = 0 : 20 \text{ lb} + 35.55 \text{ lb} - 32.96 \text{ lb} - A_H = 0$
   hence, $A_H = 22.59$ lb
3. For the vertical plane:
   $\sum M_A = 0$:
   $9.33 \text{ lb } (2 \text{ in.}) - 16.58 \text{ lb } (10 \text{ in.}) + B_v(12 \text{ in.}) = 0$
   hence, $B_v = 12.26$ lb
   $\sum F = 0 : -9.33 \text{ lb} + 16.58 \text{ lb} - 12.26 \text{ lb} + A_v = 0$
   hence, $A_v = 5.01$ lb
4. The bearing radial loads are:
   $A_{rad} = \sqrt{22.59^2 + 5.01^2} = 23.14$ lb, hence, $A_{rad} = 23.14$ lb ■

   $B_{rad} = \sqrt{32.96^2 + 12.26^2} = 35.17$ lb, hence, $B_{rad} = 35.17$ lb ■

Comments:

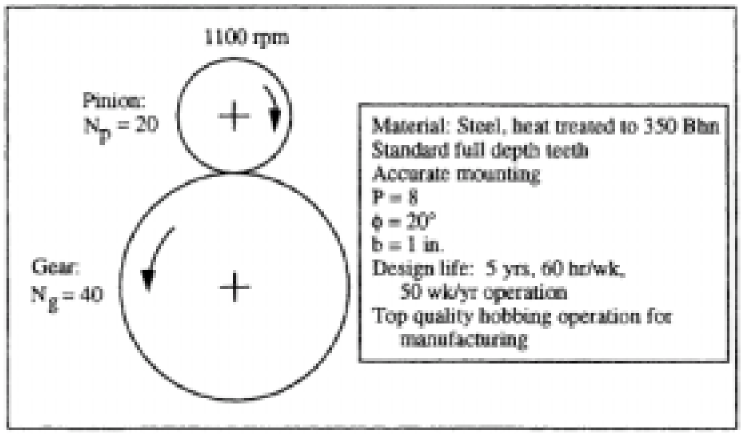
1. The effect of considering friction losses in the gears and bearings is to reduce the torque transmitted to the output shaft while keeping the speed ratios the same. A reduction in torque transmitted will result in lower gear tooth loads and hence lower radial loads on the countershaft bearings.
2. The pinion on the countershaft has higher tooth loads than the gear on the countershaft because the pinion has a smaller radius while transmitting the same torque. The higher tooth load on the pinion leads to a higher radial load on the bearing closer to the pinion.
3. The effect of choosing a smaller diametral pitch for the gears in the second reduction stage is to provide larger teeth to withstand the higher torques and tooth loads of the second stage.
4. If load sharing between teeth is considered, the transfer of gear tooth forces is not strictly at the pitch point and will lead to different radial forces in the bearings.

SOLUTION (15.29)

Known: A pair of mating spur gears of specified geometry, material and manufacturing quality is given. The pinion is driven by an electric motor of specified rpm and the gear drives a blower. Design life is specified.

Find: Determine the horsepower rating of the gear set for a safety factor of 1.5 and 99% reliability based only on bending fatigue.

Schematic and Given Data:



Assumptions:

1. The gears mesh along their pitch circles.
2. All the gear tooth loads are transmitted at the pitch point.
3. There is no loadsharing between the teeth.
4. The electric motor and blower constitute uniform load driver and driven equipment.
5. Top quality hobbing operation for manufacturing corresponds to curve C in Fig. 15.24 (to estimate velocity factor K_v).

Analysis:

1. From Eq. (15.17): $\sigma = \dfrac{F_t P}{b\,J} K_v\, K_o\, K_m$

   K_v requires finding the pitch line velocity as,

   $$V = \frac{\pi\, d\, n}{12} = \frac{\pi(20/8)1100}{12} = 720 \text{ fpm}$$

   from Curve C of Fig. 15.24,

   $$K_v = \frac{50 + \sqrt{720}}{50} = 1.54$$

   from Fig. 15.23(a), J = 0.24 (for the pinion, as it is weaker - and with no load sharing)
   Also, $K_m = 1.6$ (from Table 15.2 - probably best judgement)
   and $K_o = 1.0$ (from Table 15.1- uniform driving and driven torque)
   Therefore,

   $$\sigma = \frac{F_t(8)}{(1.0)(0.24)}(1.54)(1.0)(1.6) = 82.1\, F_t$$

2. From Eq. (15.18):
   $S_n = S_n'\, C_L\, C_G\, C_s\, k_r\, k_t\, k_{ms}$
   $= (250 \times 350)(1)(1)(0.66)(0.814)(1)(1.4)$
   $= 65{,}812$ psi
   where
   $S_n' = 250$ (Bhn) $= 250 \times 350$ psi for infinite life,
   since design life $= 5$ yr $\times$ (50 wk/yr) $\times$ (60 hr/wk) $\times$ (60 min/hr) $\times$ 1100 rpm
   $= 9.9 \times 10^8 > 10^6$ cycles
   $C_L = 1.0$,
   $C_G = 1.0$ since $P > 5$
   $C_s = 0.66$ from Fig. 8.13
   $k_r = 0.814$ from Table 15.3
   $k_t = 1$ and $k_{ms} = 1.4$ since the pinion is not an idler
3. For SF = 1.5 : $82.1(1.5\, F_t) = 65{,}812$
   hence, $F_t = 534.4$ lb

   $$\dot{W} = \frac{F_t V}{33{,}000} = \frac{(534.4)(720)}{33{,}000} = 11.66 \text{ hp}$$

   Answer : approximately 11.7 hp ■

Comments:

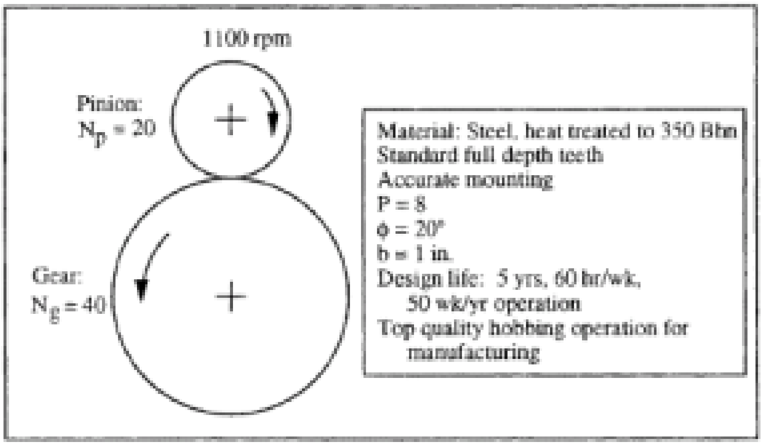
1. The bending stresses can be reduced for the specified rpm by decreasing P or increasing b. But these parameters as well as the factors K_v and J are closely interrelated. Decreasing P for the same number of teeth increases pitch diameter which leads to larger pitch line velocity and hence to larger values of K_v and σ. Decreasing P for the same pitch diameter decreases the number of teeth resulting in a smaller value of J and a larger value of σ. Increasing the value of b requires accurate mounting and manufacturing to utilize the entire face width and ultimately tends to increase the value of K_m. Thus choice of suitable values for gear geometry parameters for specific applications requires balancing the parameter values with other side effects.
2. In this problem the design life of the gear pair did not enter into the solution except to determine whether the gears were to be rated for finite or infinite life.

SOLUTION (15.35)

Known: A pair of mating spur gears of specified geometry, material and manufacturing quality is given. The pinion is driven by an electric motor of specified rpm and the gear drives a blower. Design life is specified.

Find: Determine the horsepower rating of the gear set for a safety factor of 1.5 and 99% reliability based on surface durability.

Schematic and Given Data:



Assumptions:

1. The gears mesh along their pitch circles.
2. The gear tooth loads are transmitted at the pitch point.
3. Tooth contact surfaces are approximated by cylinders.
4. Surface stresses are unaffected by lubricant and sliding friction.

Analysis:

1. From Eq. (15.24): $\sigma_H = C_p \sqrt{\dfrac{F_t}{b\, d_p\, I} K_v\, K_o\, K_m}$

   with $I = \dfrac{\sin\phi\cos\phi}{2}\dfrac{R}{R+1} = \dfrac{\sin 20^\circ \cos 20^\circ}{2} \cdot \dfrac{2}{2+1} = 0.107$

   and b = 1 in., $K_v = 1.54$, $K_o = 1.0$, $K_m = 1.6$,
   $d_p = N_p/P = (20/8)$ in. (from the analysis of Problem 15.25),

   Therefore, $\sigma_H = 2300\sqrt{\dfrac{F_t\,(1.54)(1)(1.6)}{(1.0)(20/8)(0.107)}} = 6980.5\sqrt{F_t}$

2. From Eq. (15.25): $S_H = S_{fe}\, C_{Li}\, C_R$
   $S_{fc} = 0.4\,(\text{Bhn}) - 10 \text{ ksi} = (0.4)(350) - 10 = 130$ ksi
   design life = 1100 cyl/min $\times$ 60 min/hr $\times$ 60 hr/wk
   $\times$ 50 wk/yr $\times$ 5 yr = 9.9×10^8 cycles
   hence, $C_{Li} = 0.8$
   $S_H = 130(0.8)(1) = 104$ ksi

3. For SF = 1.5 :
   $104{,}000 = 6980.5\sqrt{1.5\,F_t}$; $F_t = 148$ lb

   $\dot{W} = \dfrac{F_t V}{33{,}000} = \dfrac{148(720)}{33{,}000} = 3.23$ hp ■

   Therefore, the horsepower rating with respect to surface durability is approximately 3.2 hp.

Comment: The horsepower rating of the gear pair is much lower when analyzed with respect to surface durability than with respect to bending fatigue (Problem 15.29). With other choices of material and geometry the opposite result can also occur. This problem illustrates the need for considering both bending fatigue and surface durability in the design and analysis of gears.

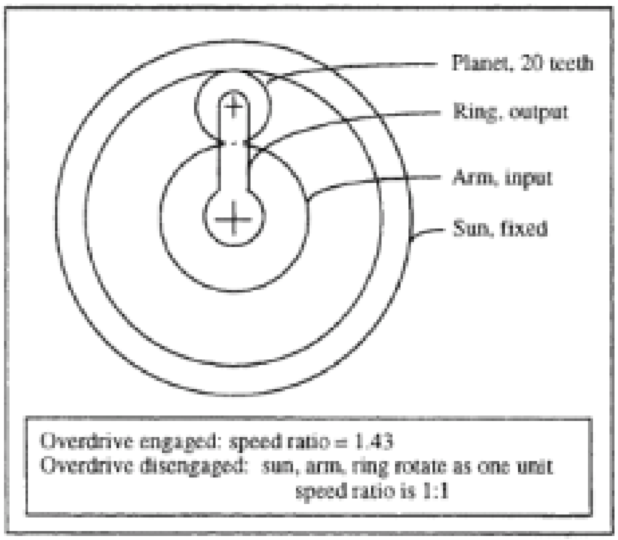
SOLUTION (15.43)

Known: A simple planetary gear train is used as an automotive overdrive unit. Speed ratio when the overdrive is engaged is specified and number of teeth on the planet is given.

Find:

(a) Determine the number of teeth on the sun and ring.
(b) Determine whether four equally spaced planets can be used.
(c) Determine whether three equally spaced planets can be used.

Schematic and Given Data:



Assumptions:

1. The gears are mounted to mesh at the pitch circles.
2. The planets are equally spaced (given).

Analysis:

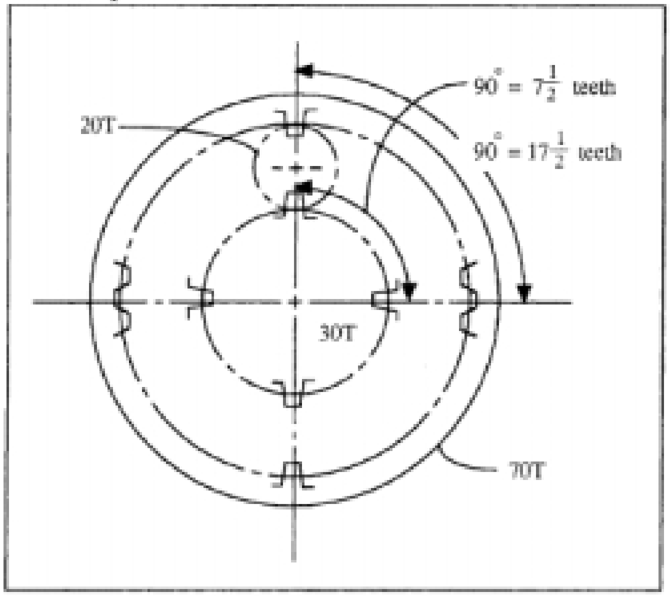
1. From Eq. (g): $1.43 = 1 + \frac{S}{R}$

   where $R = S + 2P = S + 40$

   Therefore, $0.43 = \frac{S}{S + 40}$, or $S = 30.175$, or $S = 30$ teeth ■

   $R = 70$ teeth (exact ratio $= 1 + \frac{30}{70} = 1.4286$) ■

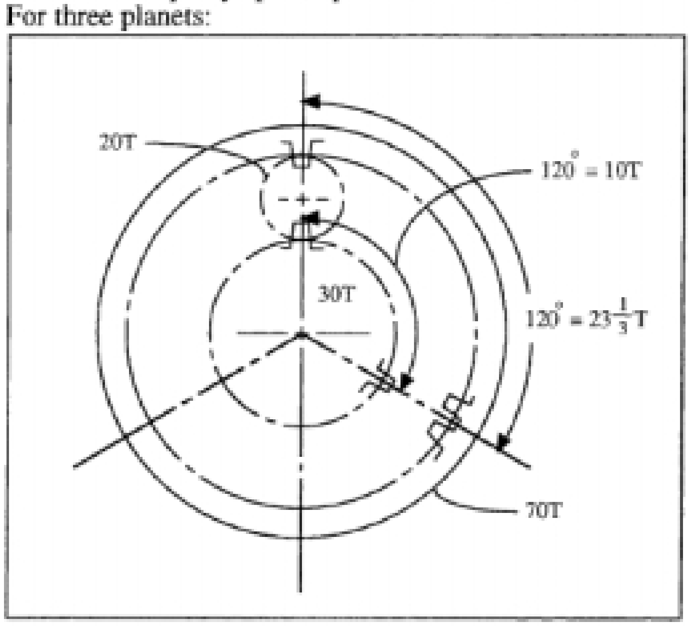
2. For four planes:



- 20 tooth planet (top position) engages with ring tooth and with sun tooth in position shown.
- Bottom position requires planet engagement with ring tooth and sun tooth; Side position requires engagement with ring space and sun space.

This works with 20 tooth (even number) planets.
Use of four equally spaced planets is workable. ■

3. For three planets:



An even-numbered planet (as 20T) will not fit the 120° position shown.
Three equally spaced planets is not workable. ■

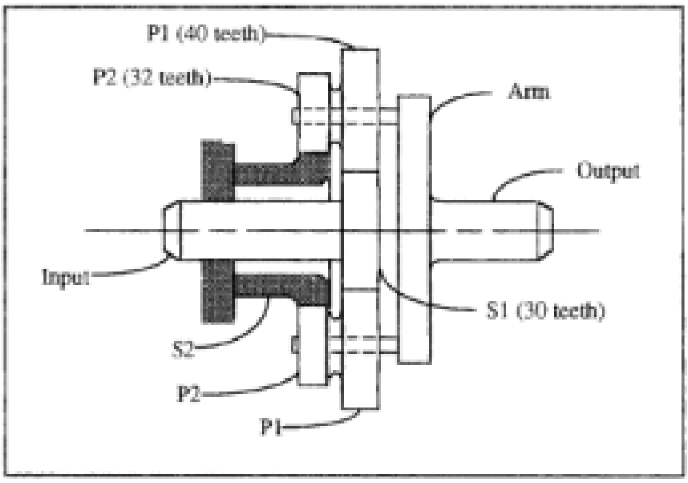
Comment: It is evident from this problem that if equally spaced planets are chosen from manufacturing and maintenance considerations, the speed ratios available are limited by geometric constraints.

SOLUTION (15.47)

Known: A planetary gear train with double planets, two suns and no ring gear is given. Numbers of teeth on the planets and one of the suns are specified. One sun is the input member, the other sun is fixed and the arm is the output member.

Find: Determine the input-output speed ratio.

Schematic and Given Data:

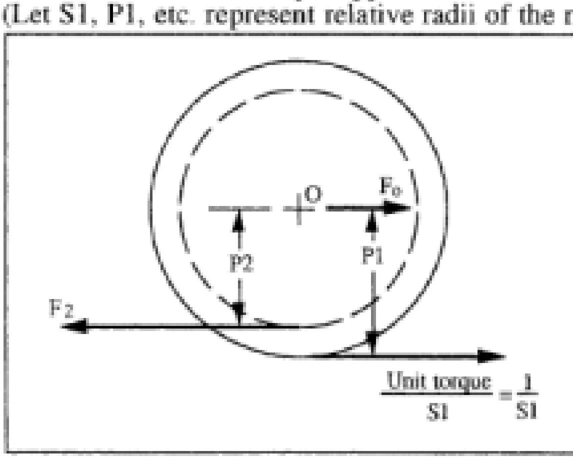


Assumption: The gears mesh along their pitch circles.

Analysis:

1. Free body analysis method

For a unit clockwise torque applied to S1, forces on the planet-pair are as follows. (Let S1, P1, etc. represent relative radii of the members):



Summing moments to zero:

$$\Sigma M_o = 0 : F_2 = \frac{1}{S1}\left(\frac{P1}{P2}\right)$$

Summing forces to zero:

$$\Sigma F = 0 : \text{hence, } F_o = \frac{1}{S1}\left(1 - \frac{P1}{P2}\right)$$

Therefore, arm torque = F_o (arm radius)

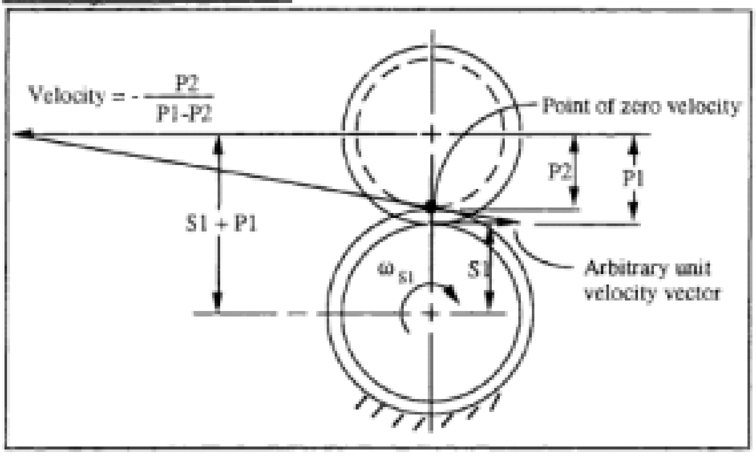
$$= \frac{1}{S1}\left(1 - \frac{P1}{P2}\right)(S1 + P1)$$

$$= \frac{1}{30}\left(1 - \frac{40}{32}\right)(70) = -0.5833$$

For 100 % efficiency, $\frac{T_A}{T_{S1}} = \frac{\omega_{S1}}{\omega_A}$

hence, $\frac{\omega_A}{\omega_{S1}} = \frac{1}{-0.5833} = -1.714$ ■

2. Velocity vector solution



We assign unit velocity, as shown in the figure.
let P1, S1, etc. represent relative radii.

From known point of zero velocity, we determine planet velocity = $-\frac{P2}{P1 - P2}$

From $\omega = V/r$:

$$\frac{\omega_A}{\omega_{S1}} = \frac{\left(-\frac{P2}{P1 - P2}\right)}{S1 + P1} \cdot \frac{S1}{1} = \frac{\left(-\frac{32}{40 - 32}\right)}{30 + 40} \cdot \frac{30}{1} = -1.714$$ ■

3. Input-output speed ratio,

$$\frac{\omega_A}{\omega_{S1}} = -1.714$$ ■

Comments:

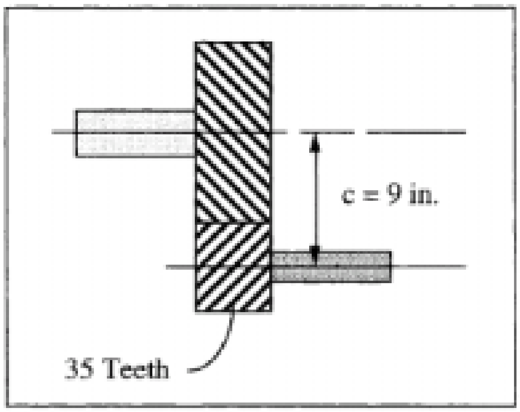
1. As this problem illustrates, it is not essential to have a ring gear to achieve speed changes in planetary gear trains. Here, the ring (an internal gear) is replaced by a second sun (an external gear) to perform the same function.
2. The use of two suns and two planets allows more flexibility in achieving speed ratio than if a ring gear were used because the pairs S1, P1 and S2, P2 can be independently chosen as long as S1 + P1 = S2 + P2.

SOLUTION (16.6)

Known: For a pair of meshing helical gears mounted on parallel shafts, the normal circular pitch, gear center distance, speed ratio and the number of teeth on the pinion are given.

Find: Determine the helix angle.

Schematic and Given Data:



Assumptions:
1. The helical gears are aligned and mounted to mesh along the pitch circles.
2. The gear teeth have standard involute profiles.

Analysis:
1. The speed ratio = 2:1, and $N_p = 35$. Hence, $N_g = 70$.
2. $m_n = p_n/\pi = 0.167$ in.
3. Center distance,

$$c = (d_g + d_p)/2 = m(N_p + N_g)/2 = \frac{m_n(N_p + N_g)}{2 \cdot \cos\psi}.$$

4. $$9 = \frac{0.167(35 + 70)}{2 \cdot \cos\psi}$$

5. Hence, $\cos\psi = 0.974$; $\psi = 13.05^\circ = 13^\circ 3'$

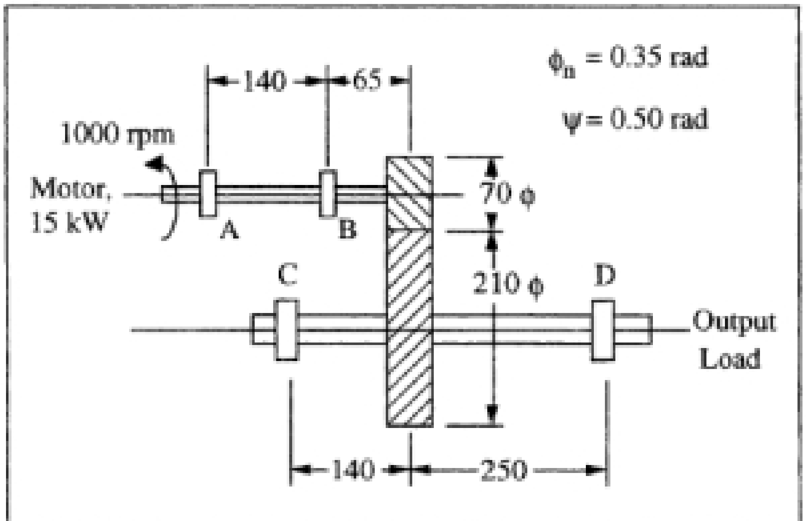
Comment: The helix angle of $13^\circ 32'$ is not in the recommended range of 15 to 30 degrees.

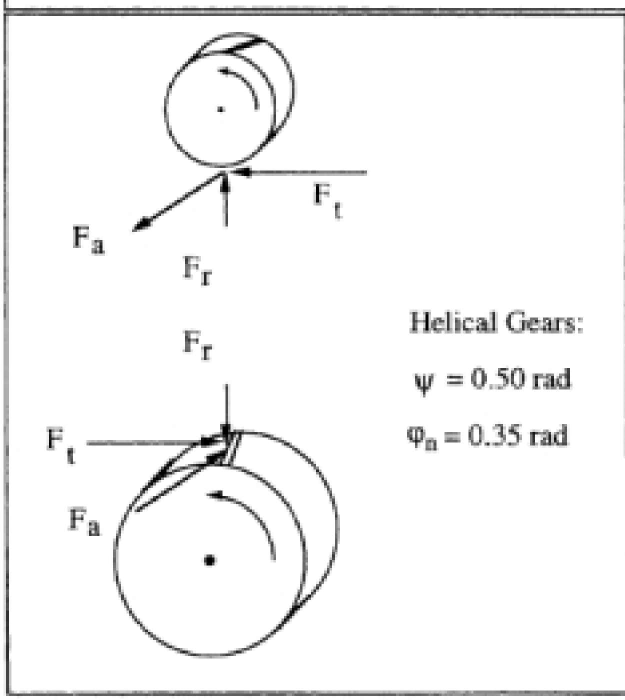
SOLUTION (16.13)

Known: A helical gear speed reducer is driven by a motor of specified speed and horsepower. Helix angle, normal pressure angle and pitch diameters of pinion and gear are given.

Find: Determine the magnitude of F_t, F_r and F_a. Show the directions of the tooth loads on the pinion and gear.

Schematic and Given Data:





Assumptions:

1. The gear teeth have a standard involute tooth profile.
2. The gears are aligned and mesh along their pitch circles.
3. All tooth loads are transferred at the pitch point and in the midplane of the gears.

Analysis:

1. From Eq. (1.2): motor torque, $T = \dfrac{9549(\dot{W})}{n} = \dfrac{9549(15)}{1000} = 143.23$ N-m

2. $F_t = \dfrac{T}{d/2} = \dfrac{143.23 \text{ N-m}}{0.035 \text{ m}} = 4092$ N ■

$$F_r = F_t \tan \phi = F_t \frac{\tan \phi_n}{\cos \psi} = 4092 \frac{\tan 0.35}{\cos 0.50} = 1702 \text{ N}$$

$$F_a = F_t \tan \psi = 4092 \tan 0.50 = 2235 \text{ N}$$ ■

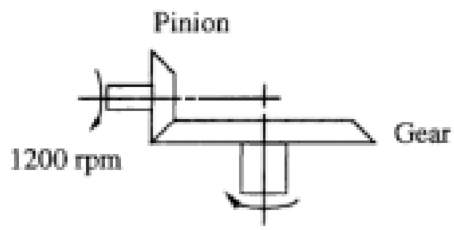
Comment: Actual loads on a tooth will be lower due to load sharing between adjacent teeth if the gears have sufficient number of teeth and good manufacturing quality.

SOLUTION (16.21)

Known: An electric motor of specified rpm drives a load involving moderate shock through a straight tooth bevel gear pair. Geometry, material and manufacturing accuracy of the gears are specified and mounting is fairly rigid with the gear straddle mounted and the pinion overhung.

Find: Estimate the horsepower that can be transmitted for a specified design life and reliability.

Schematic and Given Data:



Bevel Gears:

$N_p = 20$

$N_g = 50$

$b = 1$ in.

$\phi = 20^{\circ}$

$P = 10$

Steel pinion and gear with hardness of 300 Bhn.

Manufacturing accuracy -- Curve B, Fig. 15.23

Design Life: 7 years, 1500 hr/yr operation, 99% reliability

Assumptions:

1. The gears are mounted to mesh along their pitch cones and the teeth have standard involute profiles.
2. Tredgold's approximation is valid.
3. Surface stresses are unaffected by the presence of lubricant and sliding friction in the contact region.
4. The gear teeth have a machined finish.
5. The gears operate at a temperature below 160° F.

Analysis:

1. Surface fatigue is usually the limiting factor with through-hardened steel gears, so we check that first.

2. From Eq. (15.24): $\sigma_H = C_p \sqrt{\frac{F_t}{bd_pI} K_v K_o K_m}$ with

   $C_p = 1.23(2300) = 2829$
   $b = 1$ in. (given)
   $d_p = N_p/P = 20/10 = 2$ in.
   $I = 0.082$ (Fig. 16.15 for $N_p = 20$, $N_g = 50$)

   $V = \pi Dn = \pi\left(\frac{2.0}{12}\right) 1200 = 628.3$ fpm
   $K_v = 1.32$ (curve B of Fig. 15.24)
   $K_o = 1.25$
   $K_m = 1.2$ (estimated from Table 16.1)
   We have $\sigma_H = 9830 \sqrt{F_t}$

3. From Eq. (15.25): $S_H = S_{fe}\, C_{Li}\, C_R$
   $S_{fe} = 0.4(300) - 10 = 110$ ksi (from Table 15.5)
   For a life of 1200 rpm $\times$ 60 $\times$ 1500 $\times$ 7 = 7.56×10^8,
   Fig. 15.26 shows $C_{Li} \approx 0.82$
   For 99% reliability, $C_R = 1$
   Hence, $S_H = 110(0.82)(1) = 90.2$ ksi
   Equating: $9830 \sqrt{F_t} = 90{,}200$; hence, $F_t = 84.2$ lb
   Therefore, horsepower transmittable is
   $\dot{W} = (84.2 \text{ lb})(628.3 \text{ fpm})/33{,}000 = 1.6$ hp

4. To be sure, we check bending fatigue strength:

   From Eq. (15.17): $\sigma = \frac{F_t P}{b\,J} K_v K_o K_m$
   $J = 0.245$ (from Fig. 16.13)

   Hence, $\sigma = \frac{F_t\,(10)}{1(0.245)}(1.32)(1.25)(1.2)$:
   $\sigma = 80.8\, F_t$

5. From Eq. (15.18): $S_n = S_n'\, C_L C_G C_s\, k_r\, k_t\, k_{ms}$
   $S_n' = (250)\text{Bhn} = 250(300) = 75{,}000$ psi
   $C_L = 1$ (bending load)
   $C_G = 1$ (for $P > 5$)
   $C_s = 0.7$ (by assumption of machined finish in Fig. 8.13)
   $k_r = 0.814$
   $k_t = 1$
   $k_{ms} = 1.4$
   hence, $S_n = 59{,}829$ psi
   Equating: $80.8\, F_t = 59{,}829$ $F_t = 740$ lb
   Hence the teeth have about 9 times as much capacity in bending fatigue as in surface fatigue.

6. Therefore, horsepower that can be transmitted by the gear pair is 1.6 hp. ■

Comments:

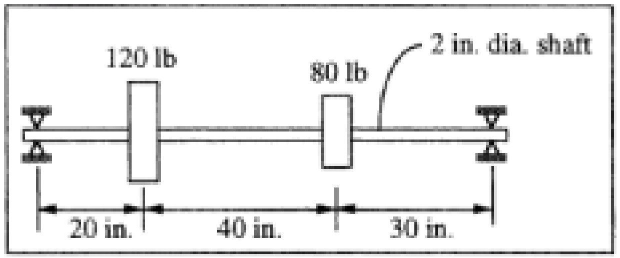
(1) To produce a balanced design of the gears, the horsepower rating for the gears based on bending fatigue and surface fatigue must be approximately equal. However, as the bending fatigue failure is catastrophic in comparison to surface fatigue failure, the factor of safety for bending fatigue failure is kept higher than that for surface fatigue failure as in this problem.

(2) Operation of the gear set for 2000 hr/yr instead of 1500 hr/yr reduces the horsepower capacity of the gear set to 1.52 hp from the original 1.6 hp. This represents only a 5% decrease in rating for a 33% increase in operating cycles. The reason for this is the characteristically low slope of the C_{Li} versus log (cycles) graph in Fig. 15.27 beyond 10^6 cycles.

SOLUTION (17.11)
Known: The dimensions of a steel shaft are given.

Find: Determine the critical speed of rotation for the steel shaft.

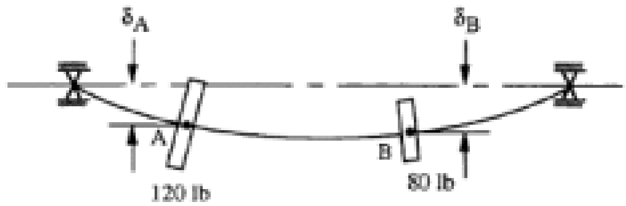
Schematic and Given Data:



Assumptions:
1. Bearing friction is negligible.
2. The bearings supporting the shafts are accurately aligned.
3. The shaft remains linearly elastic.
4. The shaft is simply supported.
5. The mass of the shaft is negligible.

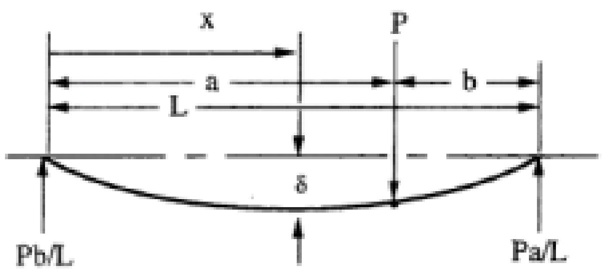
Analysis:

1.



2. Using the equation for a concentrated load at any point for a simply supported beam (Appendix D-2) and the method of superposition, the deflections δ_A and δ_B can be determined using $\delta = \dfrac{Pbx}{6LEI}(L^2 - x^2 - b^2)$ for $0 \le x \le a$

where for steel $E = 30 \times 10^6$ psi and for a round shaft $I = \dfrac{\pi d^4}{64} = \dfrac{\pi(2)^4}{64} = 0.785$ in.^{4}



Deflection at A due to 120 lb:

$$\delta = \frac{120(70)(20)}{6(90)(30 \times 10^6)(0.785)}(90^2 - 20^2 - 70^2) = 0.0370 \text{ in.}$$

Deflection at A due to 80 lb:

$$\delta = \frac{80(30)(20)}{6(90)(30 \times 10^6)(0.785)}(90^2 - 20^2 - 30^2) = 0.0257 \text{ in.}$$

Total deflection at A: $\delta_A = 0.0370 + 0.0257 = 0.0627$ in.

Deflection at B due to 120 lb:

$$\delta = \frac{120(20)(30)}{6(90)(30 \times 10^6)(0.785)}(90^2 - 30^2 - 20^2) = 0.0385 \text{ in.}$$

Deflection at B due to 80 lb:

$$\delta = \frac{80(30)(60)}{6(90)(30 \times 10^6)(0.785)}(90^2 - 60^2 - 30^2) = 0.0408 \text{ in.}$$

Total deflection at B: $\delta_B = 0.0385 + 0.0408 = 0.0793$ in.

3. Using Eq. (17.2) in Fig. 17.5(b):

$$n_c \approx \frac{30}{\pi}\sqrt{\frac{g(w_A\delta_A + w_B\delta_B)}{w_A\delta_A^2 + w_B\delta_B^2}}$$

$$n_c \approx \frac{30}{\pi}\sqrt{\frac{(32.2)(12\frac{\text{in.}}{\text{ft}})[(120)(0.0627)+(80)(0.0793)]}{120(0.0627)^2 + 80(0.0793)^2}}$$

= 708 rpm ■

SOLUTION (17.14)

Known: A helical gear mounted on a shaft is simply supported by bearing A and bearing B and has specified load components acting on it. The left end of the shaft is free, the right end, near the bearing B, is attached to a flexible coupling to transmit torque. Bearing B takes thrust. Shaft material, its ultimate and yield strength, and stress concentration factors are given.

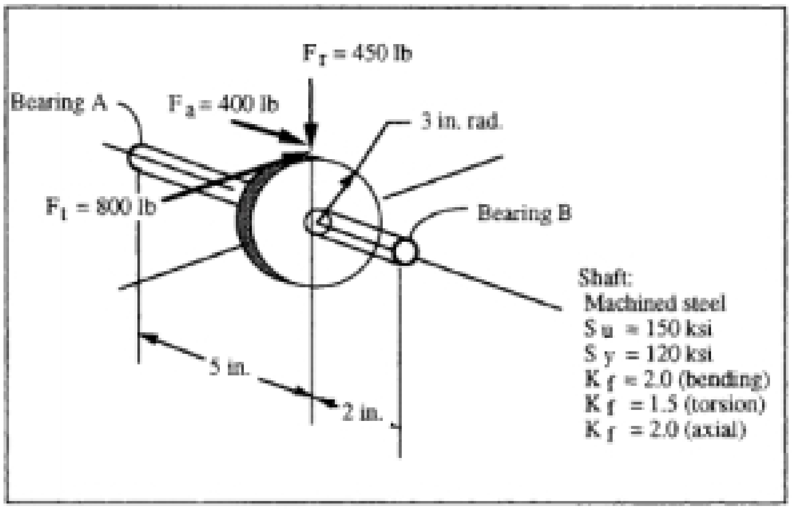
Find:

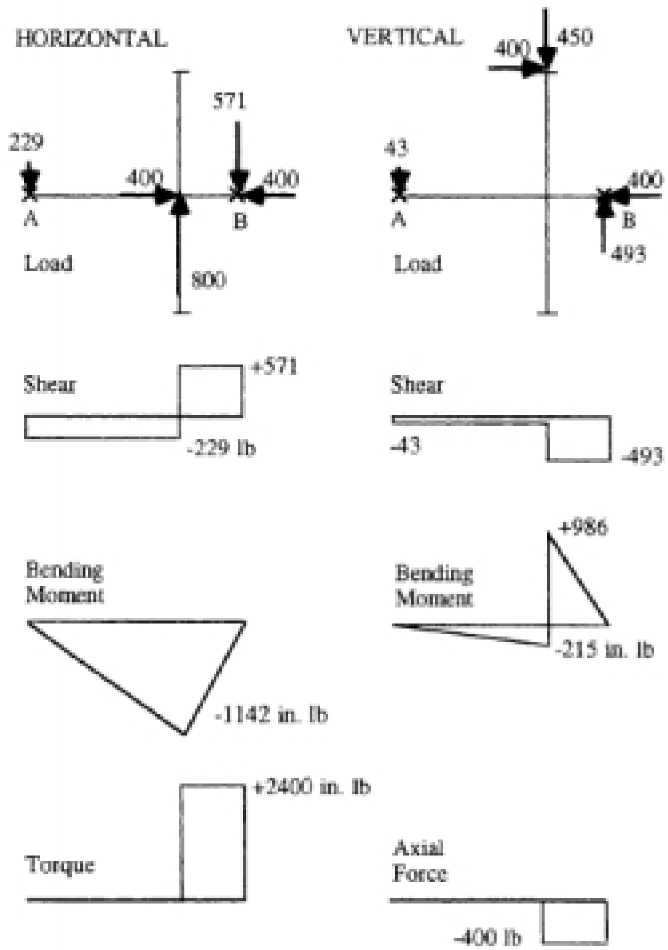
(a) Determine load, shear force and bending moment diagrams for the shaft in the vertical and horizontal planes and also diagrams for torsional and axial loading.

(b) Determine the radial and thrust loads on the bearings.

(c) Identify the critical cross section of the loaded shaft and for this location determine the cross sectional diameter required for infinite design life.

Schematic and Given Data:





Assumptions:

1. The bearing widths are small relative to the length of the shaft so that they can be idealized as point supports.
2. Bearing friction is negligible.
3. Shaft deflection is small so that locations and directions of loads are constant with respect to the shaft.
4. The gear is rigidly connected to the shaft.
5. The weights of the shaft and gear can be neglected.
6. Axial stresses are negligible compared to torsion stresses (to be verified).
7. The diameter required at the critical section is between 0.4 in. and 2 in. so that the gradient factor, $C_G = 0.9$ according to Table 8.1.

Analysis:

1. From the free body diagrams in the horizontal and vertical planes,
   $\sum M_A = 0;\ 800(5) = B_H\,(7)$
   hence, $B_H = 571$ lb
   $\sum M_A = 0;\ 450(5) + 400(3) = B_v\,(7)$
   hence, $B_v = 493$ lb
   Therefore, the loads on the bearings A and B are:
   $A_r = \sqrt{229^2 + 43^2}$: $A_r = 233$ lb
   $B_r = \sqrt{571^2 + 493^2}$: $B_r = 754.4$ lb $B_t = 400$ lb (thrust)
2. The most critical section is just to right of the gear.
   For the most critical section:

   $$\tau_m = \frac{16T}{\pi d^3} K_f = \frac{16(2400)}{\pi d^3}(1.5) = \frac{18{,}335}{d^3}$$

   $$\sigma_{a,m} \text{ (axial mean stress)} = \frac{P}{A} K_f = -\frac{400(4)}{\pi d^2}(2) = -\frac{1019}{d^2}$$

   $$\sigma_{b,a} \text{ (bending alt. stress)} = \frac{32M}{\pi d^3} K_f = \frac{32\sqrt{1142^2 + 986^2}}{\pi d^3}(2)$$

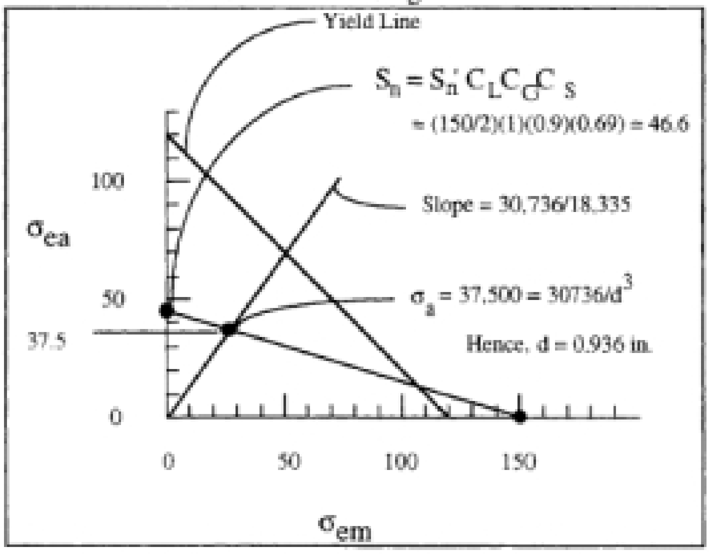
   $$\sigma_{b,a} = \frac{30{,}736}{d^3}$$
3. Applying the Fig. 8.16 procedure:

   $$\sigma_{e,a} = \sqrt{\sigma_{b,a}{}^2 + 0} = \sigma_{b,a} = \frac{30{,}736}{d^3}$$

   Assume σ_{am} is negligible, then $\sigma_{em} = 0 + \sqrt{\tau_{am}{}^2 + 0} = \tau_m = \dfrac{18{,}335}{d^3}$

   The slope of the load line is $\sigma_{ea}/\sigma_{em} = 30{,}736/18{,}335$.

4. We now construct a Goodman diagram:



5. From the Goodman diagram, d = 0.936 in. ■

6. Note: for d = 0.94, $\sigma_{am} = 1019/0.94^2 < 1.2$ ksi and is therefore negligible, and $C_G = 0.9$. Hence earlier assumptions are appropriate.

Comments:

(1) Consideration of the weight of the gear and the shaft will produce additional radial and/or thrust loads on the bearing depending on the orientation of the shaft axis in the application.

(2) If the bearing friction forces are high enough to warrant consideration, they will change the torque diagram and will reduce the maximum torque value at the critical section by the friction torque of the left bearing.

(3) Although the axial load is of the same order of magnitude as the radial load the axial stress is very much smaller than the bending stress in this case because the bending moment is fairly large. For short shafts the same axial load can cause stresses comparable in magnitude to the bending stresses since the maximum loading moment will be smaller for a shorter shaft.

SOLUTION (18.7D)

Known: A multiple-disk dry clutch transmits a specified horsepower at a specified rpm. Design values for r_i, r_o, p_{max} and f are given and materials are specified.

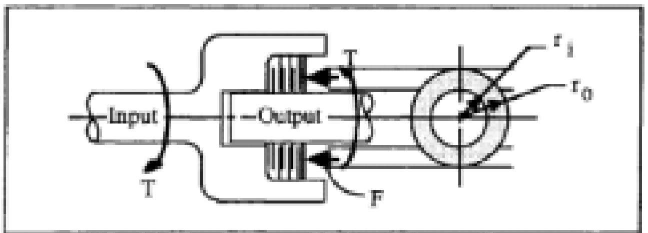
Find:

(a) Determine the total number of disks needed for a factor of safety of 2.

(b) Determine the least clamping force for (a).

(c) Determine the interface pressures at inside and outside radii after "break in".

Schematic and Given Data:



Materials: Hardened steel and sintered bronze

6 hp, 200 rpm

SF = 2

$2\,r_i = 5$ in.

$2\,r_o = 7$ in.

$p_{max} = 225$ psi

$f = 0.20$

Assumptions:
1. The friction coefficient is constant.
2. The friction in the bearings is negligible.
3. The pressure distribution corresponds to a uniform wear rate with wear proportional to pressure and rubbing velocity.

Analysis:
1. From Eq. (18.6): $T = \pi p_{max}\, r_i\, f(r_o^2 - r_i^2)N$

   Therefore, $T = \dfrac{63000\,\dot{W}}{n} = \dfrac{63000(6 \times 2)}{200} = 3780$ lb in.
   with $f = 0.2$, $p_{max} = 225$ psi, $r_i = 2.5$ in., and $r_o = 3.5$ in.:
   $3780 = \pi(225)(2.5)(0.2)(12.25 - 6.25)N$

   hence, $N = \dfrac{3780}{\pi(225)(2.5)(0.2)(6)} = 1.78$

   We use N = 2, requiring 3 disks. ■

2. From Eq. (18.7): $\mathbf{T} = Ff\left(\dfrac{r_o + r_i}{2}\right)N$:
   $3780 = F(0.2)(3)2$
   hence, $F = 3150$ lb ■
3. From Eq. (18.5): $F = 2\pi p_{max}\, r_i\,(r_o - r_i)$:
   $3150 = 2\pi p_{max}(2.5)(3.5 - 2.5)$
   hence, $p_{max} = 200.5$ psi
   Thus, pressure at inside radius = 200.5 psi and pressure at outside radius

   $= 200.5\left(\dfrac{5}{7}\right) = 143.2$ psi. ■

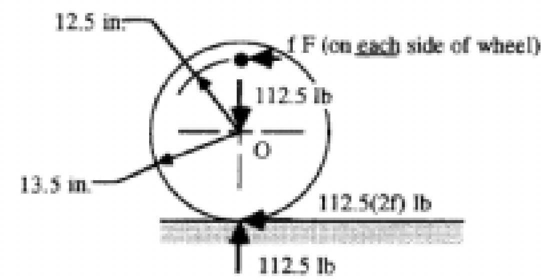
Comment: The rpm of rotation is irrelevant to the design of the clutch once torque capacity is determined since the uniform wear rate is assumed linearly proportional to rubbing velocity and rubbing velocity is linearly proportional to radius. The rpm would play a role in the design if the wear rate were a non-linear function of rubbing velocity and uniform wear rate was still expected.

SOLUTION (18.11)

Known: The rolling radius and the radius to the center of caliper disk brake pads of a bicycle are given. The combined weight of the bicycle and rider equally distributed to the two wheels is given. The ratio of friction coefficients between tire and road, and between brake pad and metal wheel rim is specified.

Find: Determine the clamping force required to slide the wheels on the road.

Schematic and Given Data:



f = coefficient of friction of brake pads

F = caliper clamping force

Assumptions:
1. The ratio of the coefficients of friction is constant.
2. The surface of the road is horizontal.
3. The friction in the wheel bearings is negligible.
4. Inertial forces can be neglected.

Analysis:

$\sum M_0 = 0$: $112.5(2f)(13.5 \text{ in.}) = 2fF(12.5 \text{ in.})$

hence, $F = 121.5$ lb ■

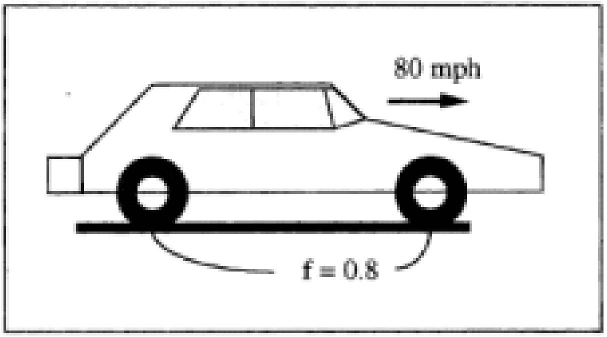
Comments:
(1) Consideration of bearing friction will reduce the clamping force necessarily by a small amount.
(2) The clamping force required will be different on a sloped road because a component of the weight will equal the normal force.

SOLUTION (18.15)

Known: A car traveling at 80 mph makes a panic stop. The brakes are applied to take full advantage of the friction coefficient of 0.8 between the tires and pavement. Acceleration produced by 80 hp engine power is determined already in Sample Problem 2.2.

Find: Determine the average horsepower absorbed by the brakes during deceleration from 80 mph to 70 mph.

Schematic and Given Data:



Assumptions:

1. The friction coefficient is constant.
2. The weight of the car is constant (as in Sample Problem 2.2) and the acceleration due to gravity is 32.2 ft/sec^2.

Analysis:

From Sample Problem 2.2, 80 hp gives 5.0 ft/s^2 acceleration at 60 mph.

By proportion, $\dfrac{hp_{braking}}{hp_{accelerating}} = \dfrac{\text{(braking friction force)(average speed during braking)}}{\text{(accelerating force)(average speed during accelerating)}}$

Therefore, the brake hp involved is: $80 \times \dfrac{32.2 \times 0.8 \text{ fps}^2}{5.0 \text{ fps}^2} \bullet \dfrac{75 \text{ mph}}{60 \text{ mph}} = 515.2$

Therefore, the horsepower involved in braking the car from 80 mph to 70 mph using the full friction force between the tires and the road is 515 hp. ■

Comments:

(1) The horsepower required for braking in a panic stop using the full friction force available from the road is more than 6 times the engine horsepower used for translational acceleration.

(2) If the wheels were braked with greater torque than allowed by the limiting friction force from the road, the wheels would begin to slide.

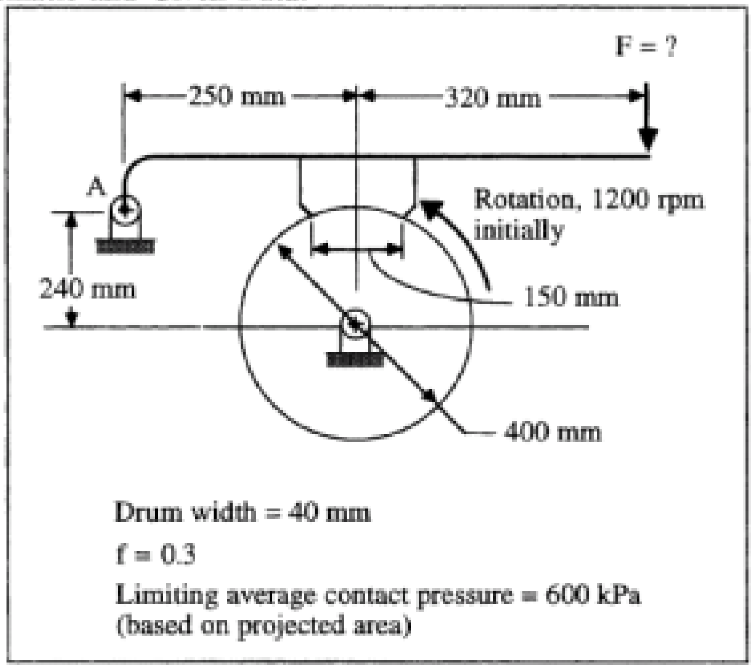
SOLUTION (18.22)

Known: A "short shoe" external drum brake has a specified drum width, friction coefficient, limiting average pressure on projected contact area, and initial drum rpm. The dimensions of the drum and lever are specified.

Find:

(a) Determine the limiting force that can be applied to the lever.
(b) Determine the braking torque.
(c) Determine whether the brake is self-energizing or de-energizing for the direction of rotation involved.
(d) Determine the radial force on the lever pivot.
(e) Determine the amount of heat generated if full brake application stops the drum in 6 seconds.
(f) Determine the average power developed by the brake during the stop.

Schematic and Given Data:



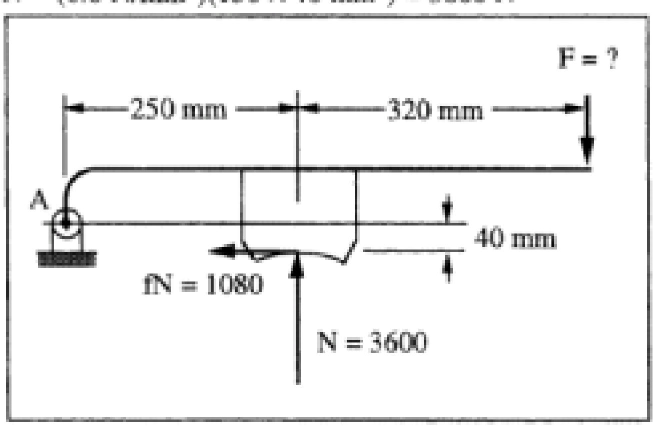
Assumptions:

1. The coefficient of friction is constant.
2. The drum is circular, mounted accurately to rotate about its center and does not wear.
3. The friction forces in the drum bearings and pivot bearings are negligible.
4. The lever is rigid and shoe displacement is small.
5. The negative acceleration due to braking is constant.

Analysis:

(a) Contact pressure = 600,000 N/m^2 = 0.6 N/mm^2

$N = (0.6\ \text{N/mm}^2)(150 \times 40\ \text{mm}^2) = 3600\ \text{N}$



$\sum M_A = 0$: $570F + 40(1080) - 3600(250) = 0$
hence, $F = 1503$ N ■

(b) Torque = 1080 N(200 mm) = 216 N·m ■

(c) Self-energizing ■

(d) Force at A $= \sqrt{(3600 - 1503)^2 + (1080)^2} = 2359$ N ■

(e) From 240 rpm to a stop in 6 sec, revs $= \dfrac{\omega_o + \omega_i}{2} t = \dfrac{240}{(60)} \cdot \dfrac{6}{2} = 12$

Work $= 2\pi T N_r = 2\pi(216 \text{ N·m})(12) = 16{,}286$ N·m
where N_r = total number of revolutions
Heat energy = 16,286 J ■

(f) From 240 rpm to a stop in 6 sec, average power = 16,286 J/(6 s) = 2714 W ■

(e*) From 1200 rpm to a stop in 6 sec, revs $= \dfrac{\omega_o + \omega_i}{2} t = \dfrac{1200}{(60)} \cdot \dfrac{6}{2} = 60$

Work $= 2\pi T N_r = 2\pi(216 \text{ N·m})(60) = 81{,}430$ N·m
where N_r = total number of revolutions
Heat energy = 81,430 J

(f*) From 1200 rpm to a stop in 6 sec, average power = 81,430 J/(6 s) = 13,572 W

Comments:

(1) The other brake shoe and lever not shown (to make up the pair of shoes and levers) will be self-energizing for the direction of rotation indicated.

(2) If the drum were not precisely cylindrical or mounted to rotate along its axis, the forces applied on the drum by the lever would directly produce torques on the drum independent of the torques due to friction.

SOLUTION (18.31)

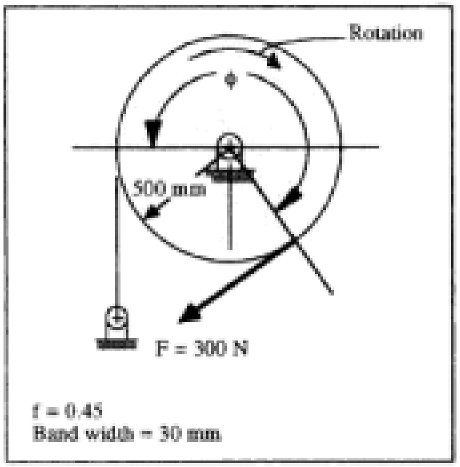
Known: A simple band brake is applied by a specified force on the band. Drum geometry and the coefficient of friction between band and drum is specified. Brake torque required is 800 N-m.

Find:

(a) Determine the necessary angle of wrap, ϕ.

(b) Determine the lining pressure in (a).

Schematic and Given Data:



Assumptions:

1. The coefficient of friction is constant.
2. The band is completely flexible laterally.
3. The full width of the band contacts the drum.
4. The pressure in the drum-band interface is uniform along the width of the drum.
5. The drum is cylindrical and mounted accurately along its axis.

Analysis:

(a) From Eq. (18.24): $T = (P_1 - P_2)r$

$800 \text{ N-m} = (P_1 - 300 \text{ N})0.5 \text{ m}$

$1600 \text{ N} = (P_1 - 300 \text{ N})$

Hence, $P_1 = 1900$ N ■

From Eq. (18.26): $\dfrac{P_1}{P_2} = e^{f\phi}$: $\dfrac{1900}{300} = e^{0.45\phi} = 6.33$

$0.45\phi = \ln 6.33 = 1.79$

Hence, $\phi = 4.10 \text{ rad} = 235^{\circ}$

(b) From Eq. (18.27):

$$p_{max} = \frac{P_1}{rb} = \frac{1900 \text{ N}}{(500 \text{ mm})(30 \text{ mm})} = 0.127 \text{ N/mm}^2$$

Hence, $p_{max} = 127$ kPa ■

Comments:

(1) The design of the band requires consideration of the tensile fatigue and flexure fatigue stresses and strengths of the band material as well as limiting contact pressures between the band and drum. Material choices are primarily influenced by the friction coefficient requirement between the band and drum.

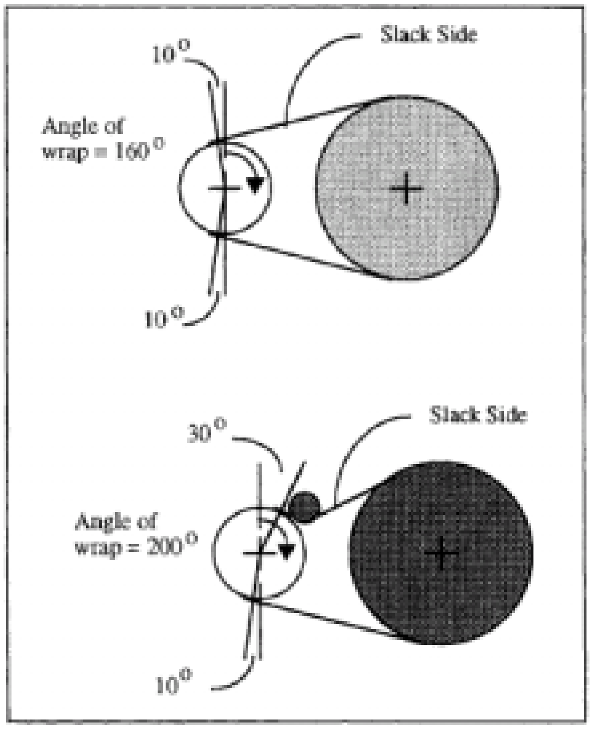
(2) If the band were not flexible enough or if the rated loads were low, a significant percentage of actuating energy could be spent in flexing the band.

SOLUTION (19.2)

Known: The angle of wrap on a pulley increases from 160° to 200° without change in slack-side tension.

Find: Determine the percentage increase in torque capacity of the pulley.

Schematic and Given Data:



Assumptions:
1. The coefficient of friction is constant throughout the angle of wrap.
2. The capacity of the belt drive is determined by the capacity of the small pulley.
3. The belt withstands the load.
4. Centrifugal force is negligible.

Analysis:
1. For $\phi = 160° = 2.79$ rad, $P_1/P_2 = e^{f\phi} = e^{(0.3)(2.79)} = 2.31$
2. For $\phi = 200° = 3.49$ rad, $P_1/P_2 = e^{f\phi} = e^{(0.3)(3.49)} = 2.85$
3. For $\phi = 160°$, $T = (P_1 - P_2)\,r = (2.31\,P_2 - P_2)\,r = 1.31\,P_2\,r$
4. For $\phi = 200°$, $T = (P_1 - P_2)\,r = (2.85\,P_2 - P_2)\,r = 1.85\,P_2\,r$

5. Increased torque capacity $= \dfrac{1.85 - 1.31}{1.31} = 0.41 = 41\%$ ■

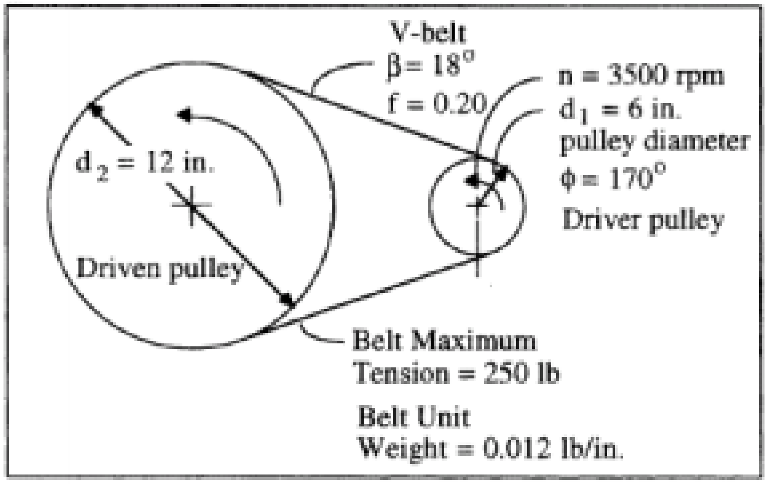
Comment: If the initial angle of wrap had been 150° rather than 160°, then the increase in torque capacity would have been 55% rather than 41%.

SOLUTION (19.10)

Known: A pulley of given rotational speed, diameter, and angle of wrap drives a V-belt with known friction, weight, and maximum tension.

Find: Determine the maximum power transmitted by the smaller pulley.

Schematic and Given Data:



Assumptions:

1. The friction coefficient is uniform throughout the contact area.
2. The effect of centrifugal force is important.

Analysis:

1. From Eq. (19.2), $P_c = m'V^2 = \left(\dfrac{0.012 \text{ lb/in.}}{(32.2 \text{ ft/s}^2)(12 \text{ in./ft})}\right)\left(\dfrac{3500}{60} \cdot 6\pi \text{ in./s}^2\right)^2$

   $= 37.5$ lb

2. From Eq. (19.3), $\dfrac{P_1 - P_c}{P_2 - P_c} = e^{f\phi}$

   where $f = \dfrac{f}{\sin\beta}$ for the V-belt

   $\dfrac{250 - 37.5}{P_2 - 37.5} = e^{(0.2/\sin 18°)(170\pi/180)} = 6.82$

   $P_2 = 68.7$ lb

3. $T = (P_1 - P_2)\, r = (250 - 68.7)\,(3) = 544$ lb in.

4. From Eq. (1.3), $\dot{W} = \dfrac{nT \text{ lb·ft}}{5252} = \dfrac{3500(544 \text{ lb in.})(1 \text{ ft}/12\text{in.})}{5252} = 30.2$ hp ■